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A Note on Learning in a Credit Economy

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Abstract

This paper studies the interaction of agents' collateral price beliefs, credit constraint and aggregate economic activity over the business cycle. Learning strengthens the role of collateral constraints in aggregate fluctuations. Under heterogeneous learning rules, numerical simulations illustrate that bankruptcy on the part of borrowers arises sooner as they track the economy faster.

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1 Introduction

This paper studies the interaction of agents' collateral price beliefs, credit constraint and aggregate economic activity over the business cycle. One strand of literature, an influential contribution is Kiyotaki and Moore (1997, henceforth KM), studies the role of credit constraints as an amplification mechanism generating large and persistent fluctuations in output and asset prices. This paper incorporates imperfect knowledge about the structure of the economy and learning about collateral prices by economic agents into KM showing that learning amplifies and propagates shocks to the economy.

The mechanism of the learning model is as follows. A positive surprise in collateral prices, say due to a positive productivity shock, brings about agents' optimism about future collateral prices. The optimism enhances borrowing capacity, which in turn boosts borrowers' collateral demand and collateral prices. The realized collateral prices reinforce agents' optimism and lead to further optimism. Associated with agents' shifting expectations and credit expansions/contractions are non-fundamental fluctuations of collateral prices and self-reinforcing rises and falls of aggregate activities.

An extension of the model considers heterogenous learning between borrowers and lenders and voluntary default of borrowers possibly arising from the divergence of borrowers' and lenders' expectations about future collateral prices. This may prevent agents from learning to form Rational Expectations (RE) which is otherwise possible under homogenous learning. Moreover, the simulation results suggest that bankruptcy on the part of the borrowers arise sooner as they track the economy faster.

A related paper is Assenza and Berardi (2009, henceforth AB) replacing the assumption of RE in the basic version of KM by adaptive learning. The paper shows that the "optimality" conditions in AB imply that agents' "optimal" choices are either suboptimal or infeasible. Moreover, it illustrates the different and correct learning dynamics.

2 The Model

The model setup is the same as that of AB presented in their section 2 except that shocks to the farmer's productivity are added. The AB model without the productivity shocks is exactly the basic version of KM except that there are some notational differences, the gatherers' production function is assigned a specific functional form $G(K_t^G) = 2\sqrt{K_t^G}$, and RE is replaced by adaptive learning. We assume homogenous expectations and learning among borrowers and lenders throughout the paper except in section 3.3.

Reproducing a few major equations from AB. The flow-of-fund constraint of the farmer (or borrower) is

$$q_t(K_t^F - K_{t-1}^F) + Rb_{t-1} + c_t^F = y_t^F + b_t \tag{1}$$

where q_t, $K_t^F - K_{t-1}^F$, b_{t-1}, c_t^F, and y_t^F are land (or collateral) price, investment in collateral holding, debt holding, consumption and production. The production function of the farmer is

$$y_t^F = (a + \epsilon_t + \bar{c})K_{t-1}^F \tag{2}$$

where $(a + \epsilon_t)K_{t-1}^F$ is the tradable part of production and $\bar{c}K_{t-1}^F$ the non-tradable part. ϵ_t is an i.i.d innovation with zero mean and constant variance.

The farmer's financing constraint is

$$b_t \leq \frac{E_t q_{t+1}}{R} K_t^F \tag{3}$$

The maximum loan he can get is the discounted and expected liquidation value of the collateral.

2.1 Optimality conditions

Some assumptions as in KM are made to ensure that the return to investment in collateral holding is sufficiently high, so the farmer prefers to borrow up to the maximum and invest in land. His optimal consumption is the nontradable part of his production

$$c_t^F = \bar{c} K_{t-1}^F \tag{4}$$

and optimal borrowing

$$b_t = \frac{E_t q_{t+1}}{R} K_t^F \tag{5}$$

Every period his inherited debt[1] is

$$b_{t-1} = \frac{E_{t-1} q_t}{R} K_{t-1}^F \tag{6}$$

Combining equations (1), (4), (5), (6) and the farmer's production function (2) delivers the farmer's land demand equation as follows

$$K_t^F = \frac{1}{\mu_t}(a + q_t + \epsilon_t - E_{t-1} q_t) K_{t-1}^F \tag{7}$$

where $\mu_t = q_t - E_t q_{t+1}/R$ is the down payment required to purchase a unit of collateral.

The gatherer's (or lender's) land demand equation is

$$\mu_t = \frac{G'(K_t^G)}{R} \tag{8}$$

where G is the production function of the gatherer. Equation (8) says that the present value of marginal product of land equals to user cost of land.

The land market clearing condition is

$$K_t^G + K_t^F = \overline{K} \tag{9}$$

Collateral prices and collateral holding process under RE are determined by equations (7), (8), and (9) given initial conditions. Under learning solving the collateral prices and collateral holding process requires additionally agents' belief updating equations and initial beliefs.

[1] (6) is assumed to hold for the initial period and it is true if initially the economy is at the steady state and agents hold RE beliefs.

2.2 Incorrect Optimality Conditions in AB

AB replace the assumption of RE in the basic version of KM by adaptive learning. This section shows that their "optimality" conditions imply that agents' "optimal" choices are either suboptimal or infeasible.

To compare with AB, the productivity shock ϵ_t is shut down in this subsection. They have a different land demand equation for the farmer, which is their equation (5) and reproduced here

$$K_t^F = \frac{a}{\mu_t} K_{t-1}^F \tag{10}$$

In contrast to equation (7), equation (10) omits capital gains/losses in land holdings $(q_t - E_{t-1}q_t)K_{t-1}^F$. Under RE equilibrium and deterministic environment, as in the KM model, agents' expectations about future land prices realize themselves due to perfect foresight. The current market value of inherited land from last period $q_t K_{t-1}^F$ offsets exactly the debt repayment $E_{t-1}q_t K_{t-1}^F$. The two collateral demand equations, i.e., equation (7) and equation (10), coincide.

However, under adaptive learning, every period agents' expectations about future collateral prices may not realize during the learning transition. The farmers may make persistent forecast errors, which produces capital gains/losses in their land holdings and additional variations in their net worth.

Implicit in AB is that the capital gains/losses are completely absorbed by consumption. This can be seen more clearly by deriving the implied consumption rule for the farmer in AB. Combining equation (1), (5), (6), (8), (10), the "optimal" consumption of the farmer implied by AB can be derived as follows

$$c_t^F = \bar{c} K_{t-1}^F + (q_t - E_{t-1}q_t)K_{t-1}^F \tag{11}$$

The implications of equation (11) are as follows. When there is capital gain in the farmer's collateral holding, i.e., $q_t K_{t-1}^F > E_{t-1}q_t K_{t-1}^F$, he will consume the capital gain instead of investing in collateral holding. He will consume part of the tradable output beyond the nontradable output, which is *suboptimal* relative to investing it, because the return to the latter is higher than consuming. When facing capital loss, i.e., $q_t K_{t-1}^F < E_{t-1}q_t K_{t-1}^F$, borrowers will consume only part of nontradable output. This implies further that they invest part of nontradable output in collateral holdings, which is *infeasible* given that they are perishable and non-tradable.

2.3 Belief Updating and the Actual Law of Motion

Appendix A shows that linearizing equations (7), (8), and (9) yields

$$q_t = \gamma_0 + \gamma_1 K_{t-1}^F + \gamma_2 E_t q_{t+1} + \gamma_3 E_{t-1} q_t - \gamma_3 \epsilon_t \tag{12}$$

$$K_t^F = \gamma_4 + \gamma_5 q_t + \gamma_6 E_t q_{t+1} \tag{13}$$

where $\gamma_0 = a$, $\gamma_1 = \frac{a^3 R^2}{2}$, $\gamma_2 = \frac{\bar{K} a^2 R^2 + 1}{2R}$, $\gamma_3 = -\frac{1}{2}[\bar{K} a^2 R^2 - 1]$, $\gamma_4 = \bar{K} - \frac{3}{(aR)^2}$, $\gamma_5 = \frac{2}{a^3 R^2}$, and $\gamma_6 = -\frac{2}{(aR)^3}$.

This model economy has the following Minimum State Variable (MSV) RE solution

$$q_t = \Psi_0 + \Psi_1 K_{t-1}^F + \Phi_q \epsilon_t \tag{14}$$

$$K_t^F = \Psi_2 + \Psi_3 K_{t-1}^F + \Phi_K \epsilon_t \tag{15}$$

where the parameters are to be determined.

In the learning model agents' Perceive Law of Motion (PLM) is

$$q_t = \phi_0 + \phi_1 K^F_{t-1} + \eta_{q,t} \quad (16)$$
$$K^F_t = \theta_0 + \theta_1 K^F_{t-1} + \eta_{K,t} \quad (17)$$

where $\eta' s$ are regression residuals. Since beliefs about θ_0 and θ_1 are irrelevant for agents' decisions, I focus on the evolution of beliefs about ϕ_0 and ϕ_1. They are revised according to

$$\phi_t = \phi_{t-1} + g_t R_t^{-1} x_{t-1}(q_{t-1} - \phi'_{t-1} x_{t-1}) \quad (18)$$
$$R_t = R_{t-1} + g_t(x_{t-1}x'_{t-1} - R_{t-1}) \quad (19)$$

where $x_t \equiv (1 \quad K^F_{t-1})'$ and $\phi_t \equiv (\phi_{0,t} \quad \phi_{1,t})'$. g_t is the gain parameter governing the speed of belief adjustments. As is standard in the literature, two gain sequences are considered: a decreasing sequence, e.g., $g_t = \frac{1}{t}$, is used to study the convergence of the learning process and a constant sequence $g_t = g > 0$, is used for numerical simulation. The latter implies that agents discount old data and give relative larger weight to recent data.

Conditional expectations are $E_t q_{t+1} = \phi_{0,t} + \phi_{1,t} K^F_t$ and $E_{t-1} q_t = \phi_{0,t-1} + \phi_{1,t-1} K^F_{t-1}$. Substituting conditional expectations into the model equations (12)-(13) delivers the Actual Law of Motion (ALM) for collateral prices and collateral holdings under learning

$$q_t = T_1(\phi_{0,t-1}, \phi_{0,t}, \phi_{1,t}) + T_2(\phi_{1,t-1}, \phi_{1,t}) K^F_{t-1} + T_3(\phi_{1,t})\epsilon_t \quad (20)$$
$$K^F_t = \frac{\gamma_4 + \gamma_6 \phi_{0,t}}{1 - \gamma_6 \phi_{1,t}} + \frac{\gamma_5}{1 - \gamma_6 \phi_{1,t}} q_t \quad (21)$$

where $T_1(\phi_{0,t}, \phi_{1,t}) = \frac{(\gamma_0 + \gamma_2 \phi_{0,t} + \gamma_3 \phi_{0,t-1}) + \gamma_2 \phi_{1,t} \frac{\gamma_4 + \gamma_6 \phi_{0,t}}{1 - \gamma_6 \phi_{1,t}}}{1 - \gamma_2 \phi_{1,t} \frac{\gamma_5}{1 - \gamma_6 \phi_{1,t}}}$, $T_2(\phi_{0,t}, \phi_{1,t}) = \frac{\gamma_1 + \gamma_3 \phi_{1,t-1}}{1 - \gamma_2 \phi_{1,t} \frac{\gamma_5}{1 - \gamma_6 \phi_{1,t}}}$ and $T_3(\phi_{1,t}) = \frac{-\gamma_3}{1 - \gamma_2 \phi_{1,t} \frac{\gamma_5}{1 - \gamma_6 \phi_{1,t}}}$.

3 Learning Dynamics

An unexpected positive impulse to farmers' productivity is considered to illustrate the learning dynamics assuming agents' initial belief is at or different from the RE belief. In addition, this section conducts E-stability analysis and studies the convergence of real time learning process under homogeneous learning among borrowers and lenders. An extension of the model considers voluntary default of borrowers assuming heterogenous learning. This section closes with some further comments on AB.

3.1 The Mechanism of the Learning Model

Numerical simulations are performed with following parameterization. The farmer's productivity is normalized at one, i.e., $a = 1$. Following KM, the parameter $\bar{K}$ is set such that farmers hold two third of the collateral in the steady state. The gross interest rate is set to 1.06. The standard deviation of the productivity shock is set to 0.00712. The functional form of the gatherer's production function implies that the collateral intensity equals to 0.5. The gain parameter is set to 0.02. This value of the gain implies that observations that are 20 years old receive a weight of $(1-0.002)^{80} \simeq 0.20$, implying agents do not discount past data too heavily. This is consistent with the literature which uses gain parameters ranging from 0.007 - 0.05-see, for example, Branch and Evans (2006), Milani (2007), and Orphanides and Williams (2005).

3.1.1 Dynamic Response

Figure 1 displays Impulse Response Functions (IRFs) of land prices, the farmer's land demand, forecasts of collateral prices and lending for 10 years under RE and learning to a 1% positive impulse to farmers' productivity assuming agents' initial belief is at the RE value. The line labeled "AB model" can be skipped for the moment and will be discussed in section 3.4.

The RE dynamics is discussed in KM and briefly reviewed here. Following a positive impulse to the farmers' productivity, their net worth and demand for collateral rise. Collateral transfers from gatherers to farmers. User cost of collateral rises and so does collateral price. Rising current collateral holdings enhance the farmers' ability to invest in following periods, which induces the persistent stay of the farmers' collateral holdings above the steady state. This reinforces the response of collateral prices when agents anticipate future user costs are above the steady state. However, after the impact period, rising user cost chokes off further rises in prices and quantities, so they converge persistently and monotonically back to the steady state.

Under learning, agents have correct forecast functions for collateral prices initially. In the impact period, responses of all variables under learning are identical to those under RE. The learning model generates additional propagation due to belief revisions and the interaction of agents' price beliefs, price realizations and credit limit. After the impact period, a positive surprise in collateral prices induces an upward belief revision and optimism about future prices relative to RE. The optimistic expectations about future collateral prices enhance borrowing capacity, so larger loans are granted by lenders. Equation (7) says that farmers can increase their collateral holdings when the capital gain is higher than the downpayment (relative to its steady state value a), i.e., $q_t - E_{t-1}q_t > \mu_t - a$. This boosts collateral prices further up. The realized collateral price reinforces agents' initial optimism and leads to further optimism. The learning model generates prolonged periods of expansions of prices and quantities. The peak responses of land prices, land holdings of farmers, collateral price forecasts, and lending under learning are 43.4%, 35.5%, 52.8%, 36.1% higher than under RE, respectively.[2]

The reversal of prices and quantities relates to the convergence of the learning process discussed in the next section. At some point the negative effect of excessive debt repayments dominates such that collateral price falls short of agents' forecast. This sets a self-reinforcing decline of prices and quantities in motion. Agents revise their beliefs downward and become pessimistic about future prices. Based on the pessimism, credit limits are tightened by the lenders, which reduces further farmers' demand on collateral. The realized prices reinforce agents' pessimism and lead to further downward adjustments of beliefs. Collateral prices and quantities oscillate around the steady state for many periods and then converge to the steady state.[3]

[2] Note in the basic version of the KM model, the steady state leverage ratio (or debt/asset ratio) equals to $1/R$ and is very large for R close to 1. A small change in collateral prices will generate large fluctuations of collateral holdings. Nevertheless, this figure can be used to illustrate the different dynamics between learning and RE.

[3] The Impulse Response Functions under learning depend continuously on the gain parameter. The larger (smaller) the gain is, the stronger (weaker) the amplification is and the more (less) likely the oscillatory dynamics happens.

A number of papers emphasized the importance of cyclical or oscillatory dynamics in RE models, e.g. Farmer and Guo (1994) and Azariadis, Bullard, and Ohanian (2004). Such dynamics is also argued as a feature of US data in these papers. Oscillatory dynamics arises in Farmer and Guo (1994) due to non-convexities and in Azariadis, Bullard, and Ohanian (2004) due to the overlapping generations structure. More closer to here, oscillatory dynamics has been found as a prominent feature of fiscal policy changes in standard Real Business Cycles models with adaptive learning in Mitra, Evans, and Honkapohja (2013).

3.1.2 Impact of Initial Beliefs

The two upper (lower) plots of figure 2 display the Impulse Response Functions (IRFs) of land prices and farmers' collateral holdings following a 1% positive productivity impulse assuming agents' initial belief is 0.1% higher or more optimistic (0.1% lower or more pessimistic) than the RE belief.[4] Relative to the RE value as initial belief, optimistic initial belief amplifies further the response. The impact response of prices and qunatities is larger than under RE, and the peak response of collateral prices and farmers' land holdings under learning is 402% and 359% higher than under RE, respectively. When agents' initial belief is pessimistic, its negative effect dominates the positive effect of the productivity impulse so that prices and quantities fall initially. After a few periods they start to increase and overshoot the steady state, with the peak response higher than that under RE. For both scenarios, prices and quantities oscillate around the steady state and converge to the steady state eventually.

3.2 E-stability and Real-time Learning

Appendix B shows that the E-stability of the REE (12)-(13) requires

$$\bar{K}a^2R^2 > \frac{2-R}{R}$$

Following KM, the gatherers' production function G is assumed to satisfy $G' > 0, G'' < 0, G'(0) > aR > G'(\overline{K})$,[5] which ensure that both farmers and gatherers are producing and holding (positive amounts of) collateral at the steady state. Given that $G(K_t^G) = 2(K_t^G)^{\frac{1}{2}}$, farmers' collateral holding at the steady state is positive implies that $K^F = \overline{K} - (aR)^{-2} > 0$. Appendix B also shows that $\bar{K}a^2R^2 > 1$ ensures the stationarity of the RE solution.

Note the coefficients in agents' PLM for land and on shocks ϵ_t will automatically converge to the RE value as long as the parameters in equation (16) converge to RE value. The following proposition summarizes the E-stability result.

Proposition 1

For all admissible parameters in $\Theta = \left\{(a, R, \bar{K}) : \bar{K}a^2R^2 > 1, R > 1\right\}$, the REE for the economy represented by equations (12) and (13) is E-stable.

Next I consider real-time learning of the REE (14)-(15). The model is mixed dating and Appendix C uses the stochastic approximation theory to prove the convergence of least-squares learning.

[4] Due to space constraints, the responses of collateral price forecasts and lending are not reported here. But their dynamics is qualitatively similar to prices and collateral holdings.

[5] See equation (5) on p. 219 of KM

Proposition 2

The REE (14)-(15) is stable under least-squares learning if the model parameters satisfy the E-stability conditions given in Proposition 1.

3.3 Heterogeneous Learning and Voluntary Default

In previous sections, homogenous expectations are assumed for all agents. This section considers heterogeneous expectations and learning among farmers and gatherers. General forms of heterogeneity are considered in the literature. For example, Honkapohja and Mirtra (2005, 2006) consider scenarios with different initial beliefs, learning algorithm, gain parameter sequences, or a mixture of them, etc, and study the convergence of adaptive learning towards RE. The paper considers a specific form of heterogeneity, i.e., different gain parameters.

The setup of this section is the same as in Section 4 and 5 of AB. Borrowers will choose to default voluntarily when their expectation about future collateral price falls below the expectation of lenders.[6] Under heterogeneous expectations, the borrowers' land demand equation is

$$K_t^F = \frac{(a + \epsilon_t + q_t - q_{t-1}^{e,G})}{\mu_t^{e,G}} K_{t-1}^F$$

The lenders' land demand equation is the same as under homogenous learning, i.e., $\mu_t^{e,G} = \frac{G'(K_t^G)}{R}$.

I explore numerically how the timing of bankruptcy decisions by borrowers depends on several key parameters of the model under heterogeneous learning. The simulation is performed as follows. The initial belief (in period 1) of borrowers and lenders is set to the RE value and the economy is assumed to start from the steady state. The lenders' gain parameter is set to 0.01. The average timing of bankruptcy of the borrowers is calculated under combinations of different gain parameters of borrowers ranging from 0.001 to 0.038 and different standard deviations of productivity shocks from 1 multiple to 1/10 of 0.00712 based on 5000 repetitions. In each repetition the timing of default is recorded as the first period when the expectation of borrowers falls below that of lenders.

Note in two scenarios borrowers will choose to default immediately, i.e., in period 2, in this model. The first scenario is a positive initial shock combined with a smaller gain parameter of borrowers. This leads to that borrowers revise their beliefs upward by less than lenders and hence have lower price expectations in the second period. The second is a negative initial shock combined with a larger gain parameter of borrowers. In this case, borrowers will revise their beliefs downward by more than those by lenders, so borrowers' expectations will fall below lenders' in the second period.[7]

Table 1 reports the average timing of default with excluding the cases of immediate default.[8]

[6] It can be shown that correcting the farmers' land demand equation does not affect their voluntary bankruptcy decisions and the bankruptcy condition in AB.

[7] In the first paragraph of page 1164 in AB, they mentioned that the farmer pays a cost C for a preliminary contract to buy a house such as in Italy but assume $C = 0$ in their paper. In a more realistic setting, all or at least most immediate defaults may be eliminated if the farmer need to pay a sufficiently large $C > 0$, because then the farmer will decide to default voluntarily when their expectation about future land price falls by more than certain positive amount below the expectation of the lender. One may conjecture that the average timing of default calculated below is an increasing function of C due to a tighter bankruptcy condition. The paper follows AB and does not pursue the case with $C > 0$.

[8] The cases of immediate default account for about half of the repetitions. The average timing of default without excluding those cases displays a qualitatively similar pattern as that with excluding them. Not surprisingly, not removing the immediate defaults will shorten the average timing of default for all parameterizations.

In Panel A (B) borrowers have a smaller (larger) gain parameter than lenders. First, the average timing of default depends negatively on the gain parameter of borrowers holding other parameters constant. Put it differently, bankruptcy arises sooner as the borrowers track the economy faster. This can be seen from each row of both panels in table 1, which reports the average timing of default varying the borrowers' gain parameter. This result is robust for different size of productivity shocks, or different gain parameters of lenders.[9] Second, the timing of default depends non-monotonically on the variance of the productivity shocks holding other parameters constant. This can be seen from each column of the table varying the size of productivity shocks.

3.4 Further Comments on AB

It appears that the E-stability condition in proposition 1 is the "same" as that in AB. However, in their proposition 1 AB claimed the condition $\bar{K}a^2R^2 > 1$ comes from the stationarity of the land process. However, proposition 1 in the paper comes from the assumptions on the production function G which ensures that both borrowers and lenders hold nonnegative amounts of collateral at the steady state omitted by AB. Moreover, the stationarity of the farmer's collateral process is satisfied automatically.

Despite the incorrectness of AB "optimality" conditions, a productivity impulse is considered to illustrate the different dynamics of the models with AB "optimality" conditions and with correct ones. Figure 1 shows that the impact response of prices and quantities under AB is much dampened, which is due to not only the smaller initial response but also consuming the capital gains instead of investment weakens the ability to reinforce agents' beliefs in subsequent periods. The price decline in AB is less prompt because the capital losses are absorbed by reducing consumption so that the farmers' net worth and collateral demand fall by less.

In the last paragraph of their page 1167, AB claimed that *"The bigger is the difference in the gain parameters and the greater the variance of the productivity shocks, the sooner bankruptcy arises."* Simulations in the previous section show that the average timing of default depends negatively on the gain parameter of borrowers instead of the gain parameter difference between borrowers and lenders and non-monotonically on the variance of the productivity shocks.[10]

4 Conclusions

The paper shows that the interaction of agents' price beliefs, price realizations, and credit limits generates interesting and realistic learning dynamics. Positive developments of fundamentals, say, due to a positive productivity impulse, lead to positive surprises in collateral price forecasts and bring about agents' optimism about future prices. The optimism enhances borrowing capacity and collateral demand and boosts collateral prices. The realized collateral prices reinforce agents' optimism and lead to further optimism. In addition, simulations show that optimistic initial belief amplifies further the effects of the productivity impulse.

[9] The results for different gain parameters of lenders/borrowers or for finer grids of the gain parameter of the borrowers are available upon request.

[10] The simulation results here suggest that their claim is not true in the learning model with correct optimality conditions. For example, comparing (1,1) element with (2,2) element of Panel A of table 1. The former (latter) reports the average timing of default when both the variance of the productivity shocks and the gain difference are larger (smaller), i.e., the standard deviation is 0.00712 (0.9*0.00712) and the gain difference is 0.009 (0.008). The average defaulting time under the former (latter) is 20.156 (20.083). This contradicts AB's claim. More such examples can be found in the table.

The paper also shows that agents can learn to form RE under homogenous least-squares learning in this credit-constrained economy. However, they may be prevented from learning the REE when agents have heterogeneous learning rule. This is because borrowers may choose to default voluntarily when their expectations fall below the lenders', which leads to the disruption of the debit/credit relationship. It is shown that bankruptcy on the part of the borrowers arises sooner as they track the economy faster.

Overoptimism is belived to have contributed significantly to the build-up of the US housing boom in 2000s and associated macro dynamics, such as rising household debt and consumption boom, etc. As an application, the current model may be extended to analyze the role of the interaction of agents' beliefs and credit constraints in producing the price boom and associated aggregate fluctuations during the housing cycle.[11]

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Appendix

A *Linearizing the Learning Economy*

Assuming that the gatherer's production function is Cobb-Douglas, i.e., $G(K_t^G) = 2\sqrt{K_t^G}$, equation (8) leads to $K_t^G = (R\mu_t)^{-2}$. The steady state of the system is $a = \mu$, $q = \frac{aR}{R-1}$, $K^G = (aR)^{-2}$ and

[11] Kuang (2013) provides such an example.

$K^F = \bar{K} - (aR)^{-2}$. I proceed to linearize the system around its steady state. Rearranging equation (7),

$$
\begin{aligned}
K^F_{t-1} &= \frac{K^F_t \mu_t}{a + q_t - E_{t-1}q_t + \epsilon_t} \\
&= \frac{(\bar{K} - \frac{1}{(R\mu_t)^2})\mu_t}{a + q_t - E_{t-1}q_t + \epsilon_t} \\
&= \frac{\bar{K}(q_t - \frac{1}{R}E_t q_{t+1})^2 - \frac{1}{R^2}}{(a + q_t - E_{t-1}q_t + \epsilon_t)(q_t - \frac{1}{R}E_t q_{t+1})} \qquad (22)
\end{aligned}
$$

Linearizing the right hand side of equation (22),

$$
\begin{aligned}
K^F_{t-1} &\simeq \bar{K} - \frac{1}{a^2R^2} + [\frac{2\bar{K}}{a} + (-\frac{\bar{K}}{a} + \frac{1}{a^3R^2}) + (-\frac{\bar{K}}{a} + \frac{1}{a^3R^2})](q_t - \frac{aR}{R-1}) \\
&\quad + [-\frac{2\bar{K}}{aR} + (\frac{\bar{K}}{aR} - \frac{1}{(aR)^3})](E_t q_{t+1} - \frac{aR}{R-1}) + [\frac{\bar{K}}{a} - \frac{1}{a^3R^2}](E_{t-1}q_t - \frac{aR}{R-1}) \\
&= \bar{K} - \frac{1}{(aR)^2} + \frac{2}{a^3R^2}(q_t - \frac{aR}{R-1}) - \frac{1}{aR}[\bar{K} + \frac{1}{(aR)^2}](E_t q_{t+1} - \frac{aR}{R-1}) \\
&\quad + \frac{1}{a}[\bar{K} - \frac{1}{a^2R^2}](E_{t-1}q_t - \frac{aR}{R-1}) + \frac{1}{a}[\bar{K} - \frac{1}{a^2R^2}]\epsilon_t
\end{aligned}
$$

Rearranging the above equation yields equation (12). Combining $K^G_t = \bar{K} - K^F_t$ and equation (8) leads to

$$
\begin{aligned}
K^F_t &= \bar{K} - (Rq_t - E_t q_{t+1})^{-2} \\
&\simeq \bar{K} - \frac{1}{(aR^2)} + \frac{2R}{(aR)^3}(q_t - \frac{aR}{R-1}) - \frac{2}{(aR)^3}(E_t q_{t+1} - \frac{aR}{R-1})
\end{aligned}
$$

Collecting the coefficients in the above equation delivers equation (13).

B *Proof of Proposition 1*

Substituting the conditional expectations into equations (12)-(13) yields the ALM for the farmer's land demand

$$
\begin{aligned}
K^F_t &= \gamma_4 + \gamma_5 q_t + \gamma_6 E_t q_{t+1} \\
&= \gamma_4 + \gamma_5 q_t + \gamma_6(\phi_0 + \phi_1 K^F_t) \\
&= \frac{\gamma_4 + \gamma_6\phi_0}{1 - \gamma_6\phi_1} + \frac{\gamma_5}{1 - \gamma_6\phi_1} q_t \qquad (23)
\end{aligned}
$$

and the ALM for collateral prices (using equation (23))

$$
\begin{aligned}
q_t &= \gamma_0 + \gamma_1 K^F_{t-1} + \gamma_2 E_t q_{t+1} + \gamma_3 E_{t-1} q_t - \gamma_3 \epsilon_t \\
&= \gamma_0 + \gamma_1 K^F_{t-1} + \gamma_2(\phi_0 + \phi_1 K^F_t) + \gamma_3(\phi_0 + \phi_1 K^F_{t-1}) - \gamma_3 \epsilon_t \\
&= T_1(\phi_0, \phi_1) + T_2(\phi_0, \phi_1) K^F_{t-1} + T_3(\phi_1)\,\epsilon_t \qquad (24)
\end{aligned}
$$

where $T_1(\phi_0,\phi_1) = \frac{(\gamma_0+\gamma_2\phi_0+\gamma_3\phi_0)+\gamma_2\phi_1\frac{\gamma_4+\gamma_6\phi_0}{1-\gamma_6\phi_1}}{1-\gamma_2\phi_1\frac{\gamma_5}{1-\gamma_6\phi_1}}$, $T_2(\phi_0,\phi_1) = \frac{\gamma_1+\gamma_3\phi_1}{1-\gamma_2\phi_1\frac{\gamma_5}{1-\gamma_6\phi_1}}$, and $T_3(\phi_1) = \frac{-\gamma_3}{1-\gamma_2\phi_{1,t}\frac{\gamma_5}{1-\gamma_6\phi_{1,t}}}$.

Combining (23) and (24) yields

$$K_t^F = \frac{\gamma_4+\gamma_6\phi_0}{1-\gamma_6\phi_1} + \frac{\gamma_5}{1-\gamma_6\phi_1}T_1(\phi_0,\phi_1) + \frac{\gamma_5 T_2(\phi_0,\phi_1)}{1-\gamma_6\phi_1}K_{t-1}^F + \frac{\gamma_5 T_3(\phi_1)}{1-\gamma_6\phi_1}\epsilon_t \tag{25}$$

At the fixed point, we have $\phi_1 = T_2(\phi_0,\phi_1) = \Psi_1$, which yields $\Psi_1 = \frac{\frac{a^3R^2}{2}}{\frac{\bar{K}a^2R^2+1}{2}-\frac{1}{R}}$. Note ϕ_0 does not appear in T_2. So only two partial derivatives, $\frac{\partial T_2}{\partial\phi_1}|_{\phi_0=\Psi_0,\phi_1=\Psi_1}$ and $\frac{\partial T_1}{\phi_0}|_{\phi_0=\Psi_0,\phi_1=\Psi_1}$, matter for the E-stability conditions. Rearranging $T_2(\phi_0,\phi_1)$

$$T_2(\phi_0,\phi_1) = \frac{\gamma_1+\gamma_3\phi_1-\gamma_1\gamma_6\phi_1-\gamma_3\gamma_6\phi_1^2}{1-\gamma_6\phi_1-\gamma_2\gamma_5\phi_1}$$

The partial derivative $\frac{\partial T_2}{\partial\phi_1}|_{\phi_0=\Psi_0,\phi_1=\Psi_1}$ is

$$\begin{aligned}
\frac{\partial T_2}{\partial\phi_1}|_{\phi_0=\Psi_0,\phi_1=\Psi_1} &= \frac{(\gamma_3-\gamma_1\gamma_6)-2\gamma_3\gamma_6\phi_1}{1-\gamma_6\phi_1-\gamma_2\gamma_5\phi_1} \\
&\quad + \frac{(\gamma_6+\gamma_2\gamma_5)[\gamma_1+\gamma_3\phi_1-\gamma_1\gamma_6\phi_1-\gamma_3\gamma_6\phi_1^2]}{(1-\gamma_6\phi_1-\gamma_2\gamma_5\phi_1)^2}|_{\phi_0=\Psi_0,\phi_1=\Psi_1} \\
&= \frac{(\gamma_3-\gamma_1\gamma_6)-2\gamma_3\gamma_6\phi_1+\phi_1(\gamma_6+\gamma_2\gamma_5)}{1-\gamma_6\phi_1-\gamma_2\gamma_5\phi_1}|_{\phi_0=\Psi_0,\phi_1=\Psi_1} \\
&= \frac{\gamma_3-\gamma_1\gamma_6-\gamma_3\gamma_6\Psi_1}{1-\gamma_6\Psi_1-\gamma_2\gamma_5\Psi_1}
\end{aligned}$$

The E-stability for the MSV RE equilibrium requires that $\frac{\partial T_2}{\partial\phi_1}|_{\phi_0=\Psi_0,\phi_1=\Psi_1} < 1$, which is equivalent to

$$\bar{K}a^2R^2 > \frac{2-R}{R}$$

The other partial derivative $\frac{\partial T_1}{\phi_0}|_{\phi_0=\Psi_0,\phi_1=\Psi_1}$ is

$$\begin{aligned}
\frac{\partial T_1}{\phi_0}|_{\phi_0=\Psi_0,\phi_1=\Psi_1} &= \frac{(\gamma_2+\gamma_3)-\gamma_3\gamma_6\phi_1}{1-(\gamma_6+\gamma_2\gamma_5)\phi_1}|_{\phi_0=\Psi_0,\phi_1=\Psi_1} \\
&= \frac{(\frac{\bar{K}a^2R^2+1}{2}-\frac{1}{R})(\frac{\bar{K}a^2R^2+1}{2}-\frac{\bar{K}a^2R^2-1}{2})-\frac{\bar{K}a^2R^2-1}{2R}}{\frac{\bar{K}a^2R^2+1}{2}-\frac{1}{R}-\frac{\bar{K}a^2R^2-1}{2R}}
\end{aligned}$$

The E-stability for the MSV RE equilibrium requires that $\frac{\partial T_1}{\phi_0}|_{\phi_0=\Psi_0,\phi_1=\Psi_1} < 1$, which is equivalent to

$$\bar{K}a^2R^2 > \frac{2-R}{R}$$

To sum up, the E-stability for the MSV RE equilibrium requires that $\bar{K}a^2R^2 > \frac{2-R}{R}$.

The stationarity of the borrower's collateral holding process (25) at REE requires

$$\left|\frac{\gamma_5\Psi_1}{1-\gamma_6\Psi_1}\right| < 1$$

which is equivalent to $\bar{K}a^2R^2 > 1$.

C *Proof of Proposition 2*

The model is mixed-dating, similar to the model of Adam, Evans, and Honkapohja (2006) which study the convergence of the real time learning process. The proof here follows them. Note agents' belief about the parameters in the evolution of farmers' collateral holdings, i.e., θ_0 and θ_1, does not appear in the ALM under learning. The proof focuses on the evolution of agents' belief about the parameters in the PLM for collateral prices. They are updated according to

$$\begin{pmatrix}\phi_{0,t}\\ \phi_{1,t}\end{pmatrix} = \begin{pmatrix}\phi_{0,t-1}\\ \phi_{1,t-1}\end{pmatrix} + g_t S_{t-1}^{-1}\begin{pmatrix}1\\ K_{t-2}^F\end{pmatrix}(q_{t-1} - \phi_{0,t-1} - \phi_{1,t-1}K_{t-2}^F) \tag{26}$$

and

$$\begin{pmatrix}\phi_{0,t-1}\\ \phi_{1,t-1}\end{pmatrix} = \begin{pmatrix}\phi_{0,t-2}\\ \phi_{1,t-2}\end{pmatrix} + g_t\left(\frac{g_{t-1}}{g_t}\right) S_{t-1}^{-1}\begin{pmatrix}1\\ K_{t-3}^F\end{pmatrix}(q_{t-2} - \phi_{0,t-2} - \phi_{1,t-2}K_{t-3}^F) \tag{27}$$

for the periods t and $t-1$

A timing change is made, i.e., $S_t = R_{t+1}$, so the moment matrix evolves according to

$$S_t = S_{t-1} + \ g_t\left(\frac{g_{t+1}}{g_t}\right)\left(x_{t-1}x_{t-1}' - R_{t-1}\right) \tag{28}$$

and

$$S_{t-1} = S_{t-2} + \ g_t\left(x_{t-1}x_{t-1}' - R_{t-1}\right) \tag{29}$$

where $g_t = \frac{1}{t}$ and $x_t = (1 \quad K_{t-1}^F)'$.

To write the entire system as a Stochastic Approximation Algorithm (SRA), I next define $\alpha_t = (\phi_{0,t}, \phi_{1,t}, \phi_{0,t-1}, \phi_{1,t-1})'$,

$$B_t = \begin{pmatrix}\alpha_t\\ vec\ S_t\\ vec\ S_{t-1}\end{pmatrix} \text{ and } X_t = \begin{pmatrix}K_{t-1}^F\\ K_{t-2}^F\\ K_{t-3}^F\\ 1\end{pmatrix}$$

With this notation the equations for parameter updating are in the standard form

$$\alpha_t = \alpha_{t-1} + g_t Q(t, \alpha_{t-1}, X_t) \tag{30}$$

where the function $Q(t, \alpha_{t-1}, X_t)$ is defined by (26)-(29). The ALM for farmers' collateral holdings can be expressed in terms of general functional notation as

$$K_t^F = C_1(\alpha_t) + C_2(\alpha_t)K_{t-1}^F + C_3(\alpha_t)\epsilon_t$$

The state vector X_t evolves according to

$$\begin{pmatrix}K_{t-1}^F\\ K_{t-2}^F\\ K_{t-3}^F\\ 1\end{pmatrix} = \begin{pmatrix}C_2(\alpha_{t-1}) & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}K_{t-2}^F\\ K_{t-3}^F\\ K_{t-4}^F\\ 1\end{pmatrix} + \begin{pmatrix}C_1(\alpha_{t-1}) & C_3(\alpha_{t-1})\\ 0 & 0\\ 0 & 0\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ \epsilon_{t-1}\end{pmatrix}$$

or

$$X_t = A(\alpha_{t-1})X_{t-1} + B(\alpha_{t-1})v_t \tag{31}$$

where $v_t = (1\ \epsilon_{t-1})'$.

System (30) and (31) is a standard form for SRAs. The associated ODEs for the model are

$$\begin{aligned}
\frac{d}{d\tau}\begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix} &= S^{-1}M(\alpha)\begin{pmatrix}T_1(\alpha)-\phi_0\\ T_2(\alpha)-\phi_1\end{pmatrix}\\
\frac{dS}{d\tau} &= M(\alpha)-S\\
\frac{d}{d\tau}\begin{pmatrix}\phi_0^1\\ \phi_1^1\end{pmatrix} &= S^{-1}M(\alpha)\begin{pmatrix}T_1(\alpha)-\phi_0^1\\ T_2(\alpha)-\phi_1^1\end{pmatrix}\\
\frac{dS^1}{d\tau} &= M(\alpha)-S^1
\end{aligned}$$

where the temporary notation of variables with/without the superscript refers to the t and $t-1$ dating in system (26)-(29) and $M(\alpha)$ data moment matrix when beliefs are fixed at α. A variant of standard arguments shows that stability of the ODE is governed by the stability of the smaller ODE

$$\frac{d}{d\tau}\begin{pmatrix}\phi_0\\ \phi_1\\ \phi_0^1\\ \phi_1^1\end{pmatrix} = \begin{pmatrix}T_1(\alpha)-\phi_0\\ T_2(\alpha)-\phi_1\\ T_1(\alpha)-\phi_0^1\\ T_2(\alpha)-\phi_1^1\end{pmatrix} \tag{32}$$

Denote α^* the fixed point or the RE belief. The Jacobian matrix of (32) evaluated at the fixed point α^* is

$$\begin{pmatrix}
\frac{\partial T_1}{\partial \phi_0} & \frac{\partial T_1}{\partial \phi_1} & \frac{\partial T_1}{\partial \phi_0^1} & 0\\
0 & \frac{\partial T_2}{\partial \phi_1} & 0 & \frac{\partial T_2}{\partial \phi_1^1}\\
\frac{\partial T_1}{\partial \phi_0} & \frac{\partial T_1}{\partial \phi_1} & \frac{\partial T_1}{\partial \phi_0^1} & 0\\
0 & \frac{\partial T_2}{\partial \phi_1} & 0 & \frac{\partial T_1}{\partial \phi_1^1}
\end{pmatrix}_{|\alpha=\alpha^*}$$

It can be shown that two eigenvalues of the matrix are zeros and the rest two are $\left(\frac{\partial T_2}{\partial \phi_1} + \frac{\partial T_1}{\partial \phi_1^1}\right)|_{\alpha=\alpha^*}$ and $\left(\frac{\partial T_2}{\partial \phi_0} + \frac{\partial T_1}{\partial \phi_0^1}\right)|_{\alpha=\alpha^*}$. Note $\left(\frac{\partial T_2}{\partial \phi_1} + \frac{\partial T_1}{\partial \phi_1^1}\right)|_{\alpha=\alpha^*} < 1$ and $\left(\frac{\partial T_2}{\partial \phi_0} + \frac{\partial T_1}{\partial \phi_0^1}\right)|_{\alpha=\alpha^*} < 1$ are exactly the E-stability conditions, hence we have proposition 2.

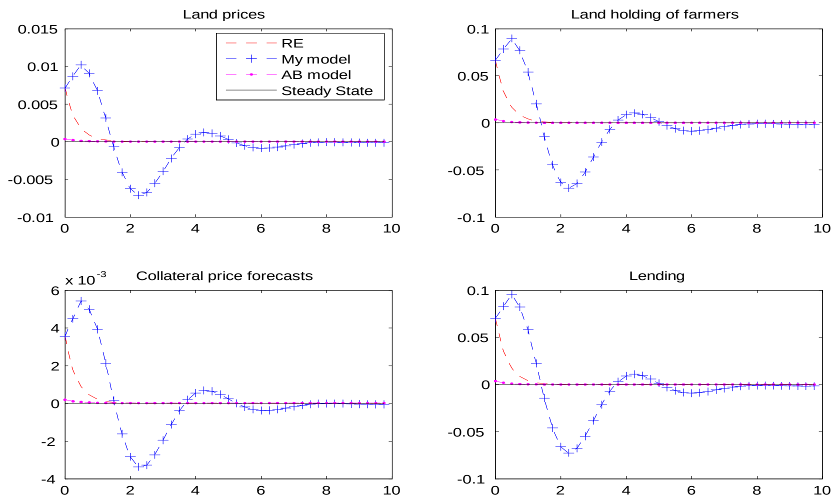

Figure 1: Response to 1% positive productivity impulse

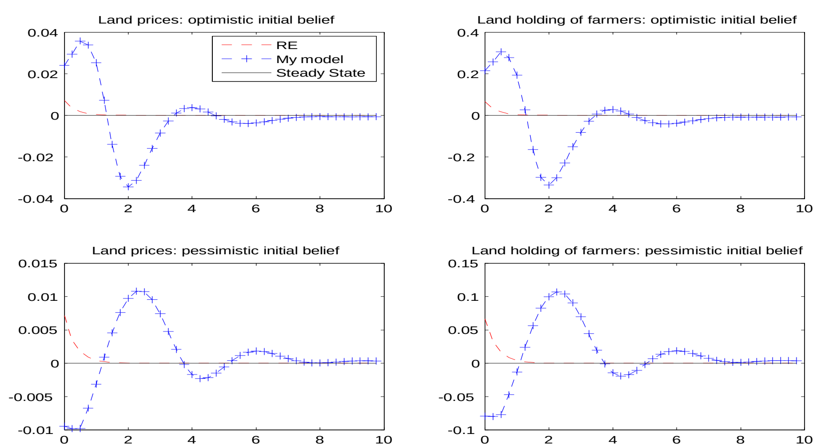

Figure 2: Response to 1% positive productivity impulse: different initial beliefs

Table 1: Dependence of the average timing of default of borrowers on borrowers' gain parameter and the size of the productivity shocks

Panel A: Borrowers have smaller gain parameters

STD				Borrowers'	gain	parameter			
	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
1	20.156	20.151	20.123	20.058	20.055	20.052	20.021	20.006	19.999
0.9	20.112	20.083	20.078	20.063	20.050	20.036	20.029	20.010	20.005
0.8	20.067	20.065	20.049	20.043	20.040	20.029	20.007	19.895	19.893
0.7	20.073	20.064	20.033	20.027	19.909	19.897	19.889	19.888	19.865
0.6	20.051	19.968	19.908	19.901	19.890	19.874	19.871	19.861	19.859
0.5	19.975	19.946	19.927	19.924	19.901	19.880	19.869	19.805	19.795
0.4	20.002	19.949	19.925	19.905	19.880	19.871	19.854	19.815	19.698
0.3	20.003	19.997	19.979	19.938	19.924	19.727	19.722	19.714	19.659
0.2	19.954	19.896	19.759	19.737	19.714	19.678	19.664	19.658	19.640
0.1	19.831	19.810	19.771	19.762	19.753	19.717	19.715	19.650	19.638

Panel B: Borrowers have larger gain parameters

STD				Borrowers'	gain	parameter				
	0.011	0.014	0.017	0.020	0.023	0.026	0.029	0.032	0.035	0.038
1	18.070	18.008	17.951	17.900	17.848	17.823	17.727	17.678	17.645	17.502
0.9	18.117	18.051	18.023	17.964	17.919	17.862	17.837	17.827	17.783	17.746
0.8	18.212	18.180	18.089	18.003	17.989	17.907	17.871	17.846	17.825	17.799
0.7	18.278	18.198	18.178	18.162	18.102	17.937	17.801	17.762	17.741	17.735
0.6	18.311	18.241	18.217	18.096	18.078	18.008	17.917	17.888	17.769	17.756
0.5	18.293	18.262	18.175	18.148	18.137	18.061	17.981	17.926	17.919	17.788
0.4	18.394	18.346	18.325	18.185	18.144	18.042	17.933	17.905	17.781	17.768
0.3	18.435	18.321	18.257	18.245	18.163	18.085	18.044	17.823	17.798	17.767
0.2	18.629	18.423	18.380	18.253	18.174	18.149	18.022	17.890	17.770	17.733
0.1	18.638	18.600	18.469	18.385	18.321	18.131	18.051	17.972	17.842	17.808

The average timing of default is calculated for different combinations of borrowers' gain parameter and different sizes of the productivity shocks based on 5000 repetitions. The lenders' gain parameter is set to 0.01. The standard deviation of the productivity shocks (column STD) ranges from 0.1 to 1 multiple of 0.00712. For each repetition, the timing of default is recorded as the first period when the expectation of borrowers falls below that of lenders.

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