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2016-09



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Securitization and Aggregate Investment Efficiency*

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May 10, 2016

Abstract

This paper studies the welfare properties of competitive equilibria in an economy with incomplete markets subject to idiosyncratic and aggregate shocks. We focus on the role of securitization, whereby borrowers can reduce idiosyncratic asset risk, which enables increased leverage and investment. In the absence of frictions in the securitization process, we show that the ability to securitize assets completes markets. When there are frictions in the market for securitized assets, requiring originators to hold some skin-in-the-game, markets remain incomplete and risk-sharing is limited. In this case, fire-sales are required to repay debt and finance new investments when the economy is hit by a negative shock. Moreover, the equilibrium may be constrained inefficient due to the existence of a pecuniary externality that can result in over or under-investment. We examine policies to correct over-investment and find that a leverage ratio restriction generates a Pareto improvement, while forcing originators to hold additional skin-in-the-game reduces welfare. Both policies reduce leverage and raise prices in a fire-sale, however tightening skin-in-the-game also directly reduces the resources available to those who most need them, which dominates the positive effect of higher prices.

JEL codes: D52, D53, E44, G18, G23.

Keywords: Securitization, pecuniary externalities, collateral constraints, financial frictions, macroprudential regulation, fire-sales, incomplete markets.

*We are grateful to Viral Acharya, Darrell Duffie, John Fender, Mike Hoy, Charles Goodhart, Nobuhiro Kiyotaki, and Frank Strobel for helpful comments and suggestions.

1 Introduction

The tremendous growth in securitization over the last several decades has significantly altered global capital markets and the nature of financial intermediation. With the rise of the originate-to-distribute model, U.S. asset backed securities outstanding have grown from approximately \$11.3 billion in 1986 to over \$2,150 billion in 2010.¹ Despite the size and importance of this transformation, there is relatively little theoretical analysis on the aggregate and welfare implications of securitization. This paper attempts to fill this gap.

In an economy with financial frictions, securitization may be valuable because it can enhance risk-sharing opportunities by substituting for missing markets. Specifically, when asset returns are subject to idiosyncratic risk and markets for hedging such risk are limited or nonexistent, pooling assets via securitization provides additional insurance. As a result, securitization can raise aggregate investment and leverage as borrowers are able to issue safer liabilities to risk-averse lenders. However, as has been shown in the literature, competitive equilibria may exhibit socially excessive aggregate investment when markets are incomplete.² Thus, while securitization enhances risk-sharing it may amplify over-investment, and as such the welfare implications of securitization may be ambiguous.³

To analyze the welfare implications of securitization we develop a dynamic general equilibrium model of investment and securitization with incomplete markets. When securitization is an imperfect substitute for missing markets, such that risk-sharing is limited by a “skin-in-the-game” constraint on sellers forcing them to retain a certain fraction of their own investments, we show that competitive equilibria may be characterized by socially excessive investment and leverage. In this case, a planner facing the same constraints as the private market can engineer a Pareto improvement by reducing financial sector leverage. The improvement arises from an increase in the price of assets in the event of a fire-sale, which

¹These figures are taken from Table 1 in [Gorton and Metrick \(2012\)](#). For a summary of the growth of securitization markets, as well as a discussion of many of the relevant issues see [Gorton and Metrick \(2012\)](#) and [Segoviano, Jones, Lindner, and Blankheim \(2015\)](#).

²For example, see [Lorenzoni \(2008\)](#).

³Generally, additional markets need not increase welfare, as first shown in [Hart \(1975\)](#).

transfers resources to individuals that have the most productive use for them. Surprisingly, requiring sellers to hold more skin-in-the-game, which also reduces leverage (indirectly) and raises prices in a fire-sale, is *always* welfare reducing.

The paper develops a three-period model of investment. In period 0, risk-neutral borrowers with limited capital, who we refer to as intermediaries, obtain funds from risk-averse investors to finance investment projects. Investments are either successful early (period 1), successful late (period 2), or fail late and return nothing. While ex-ante homogeneous, intermediaries differ at period 1 as returns on their individual investments may arrive early or late. Those with early returns (early types) will have sufficient funds to meet their debt obligations and to invest in new opportunities that arrive in period 1. Intermediaries that do not have early returns (late types) will be required to raise funds. Importantly, financial frictions rule out state-contingent contracts at period 0 and borrowing at period 1. As a result, late types will sell assets to early types via a spot market to generate funds to service debt and invest in new opportunities. However, late types may be constrained in their ability to raise funds, forgoing positive NPV investments and creating a “credit crunch.” The extent to which late borrowers are constrained is dependent on the prevailing asset price, which in turn depends on the aggregate funds early types have and the aggregate quantity of assets for sale. Crucially, atomistic intermediaries do not anticipate the impact of their period 0 investment and securitization decisions on the price of assets at period 1. Therefore, a pecuniary externality arises that can result in period 0 investment that is either insufficient or excessive from a social perspective.

In our model, securitization mitigates financial frictions by moving funds from early to late types at period 1, substituting for contingent contracts that would provide such transfers. This allows intermediaries to create more safe debt for risk-averse investors by increasing pledgeable income when returns are late. In other words, demand for securitization is driven by the need for high-quality collateral required to borrow from risk-averse investors. This is a standard partial equilibrium view of how more securitization can lead to increased leverage

and investment.⁴ However, our framework also highlights a novel aspect of securitization; that securitization affects spot market prices by changing the distribution of cash in the market. Specifically, with more securitization at period 0, demand for assets at time 1 declines since the funds of early types are reduced. On the supply side, late types require less funds and have more assets to sell. This results in a reduction in the price of assets at period 1, creating a transfer from late to early types and thereby amplifying the pecuniary externality.

Our framework allows us to shed new light on the welfare implications of policies to curb excessive leverage in the financial sector. We show that investment is generally inefficient when securitization is not complete and that leverage restrictions, akin to those outlined in Basel III, can be welfare improving when the competitive equilibrium is characterized by too much investment. However, implementing leverage restrictions presents a number of practical difficulties due to asymmetric information between intermediaries and regulators.⁵

Alternatively, it seems plausible that recent policies designed to restrict the extent of securitization in the financial system could also increase welfare, since securitization affects leverage indirectly. For example, the retention requirements in the U.S. Dodd-Frank Act and the European Capital Requirements Directive.⁶ Indirect policies such as these are important from a practical perspective because they require significantly less information than direct restrictions on the balance sheet, such as capital or leverage constraints.⁷ Perhaps more importantly, market restrictions on skin-in-the-game could influence the leverage of non-regulated institutions, since securitized lending is at the heart of so-called “shadow banking.” We show that the total effect of forcing more skin-in-the-game can be decomposed into

⁴See for example [DeMarzo \(2005\)](#), [Coval, Jurek, and Stafford \(2009\)](#), [Gorton and Metrick \(2012\)](#), and [Kiff and Kisser \(2014\)](#).

⁵For example, differences in the accounting treatment of derivatives (GAAP versus IFRS) imply very different levels of leverage for large banks.

⁶Specifically Article 122a of the European Capital Requirements Directive and Section 941 of the U.S. Dodd-Frank Wall Street Reform and Consumer Protection Act both require a five percent minimum retention rate by securitizers or originators, with exceptions for various types of underlying assets. Notably, “qualified” residential mortgage backed securities, which are backed by loans that meet a specific underwriting criteria.

⁷In a general framework with pecuniary externalities, [Kilenthong and Townsend \(2016\)](#) consider how to implement constrained-efficient outcomes via alternative market-based mechanisms.

a direct effect and a price effect. There is a direct tightening of the constraints on late intermediaries, which reduces the collateral value of investments, causing them to reduce leverage ex-ante. On the other hand, reduced aggregate investment increases the price of assets in a fire-sale, which increases the collateral value of assets and results in increased leverage. The direct effect is obvious and provides an intuitive rationale for tightening constraints as a means to reduce excessive investment. However, this is undone by the price effect, leaving the negative impact of the tighter constraint to dominate. This makes clear that the rationale for policies to increase skin-in-the-game as a means to reduce excessive leverage cannot rely solely on partial equilibrium arguments.

For our results to obtain we require market incompleteness coupled with frictions in the securitization process. The missing markets we assume are a precondition for securitized lending, since the existence of contingent securities at period 0 or frictionless borrowing at period 1 eliminates the value in securitizing assets in our model. While we are agnostic about the specific market failure(s) that result in borrowing constraints, the literature has highlighted a number of possibilities. For example, limits to borrowing may be justified by the presence of asymmetric information as in [Stiglitz and Weiss \(1981\)](#), limited commitment following [Kehoe and Levine \(1993\)](#), or moral hazard as in [Holmström and Tirole \(1997\)](#). The assumption that issuers of securitized assets may be forced to hold some skin-in-the-game is also vital. The reasons for this are not modeled in the paper, but can be motivated by the existence of an informational asymmetry between originators of securities and outsiders. This type of argument is made formal in [DeMarzo and Duffie \(1999\)](#), who find that issuers holding skin-in-the-game is an optimal contractual arrangement in the presence of issuer private information. This can arise in other interpretations, such as [Holmström and Tirole \(1997\)](#), where this type of structure arises to address moral hazard.⁸ This assumption can also be supported by new regulations (discussed above) that force issuers of securitized assets

⁸[Cerasi and Rochet \(2014\)](#) provide a model of securitization of this type in which banks hold an equity tranche to maintain proper incentives. However, when the initial investment need not be raised in conjunction with securitization, the skin-in-the game requirement need not be part of the optimal contract as shown by [Hartman-Glaser, Piskorski, and Tchisty \(2012\)](#).

to retain economic exposure in an effort to ensure their interests align with investors.

Related Literature

This paper is related to the study of pecuniary externalities which arise from incomplete markets. This literature goes back to the seminal work of [Hart \(1975\)](#), [Diamond \(1980\)](#), [Stiglitz \(1982\)](#), [Geanakoplos and Polemarchakis \(1986\)](#) and [Greenwald and Stiglitz \(1986\)](#). In our application, market incompleteness precludes individuals from equalizing marginal returns to investment. This is similar to the type of friction studied in [Shleifer and Vishny \(1992\)](#), [Gromb and Vayanos \(2002\)](#), [Caballero and Krishnamurthy \(2001\)](#), [Allen and Gale \(2004\)](#), [Lorenzoni \(2008\)](#), [Farhi, Golosov, and Tsyvinski \(2009\)](#), [Davila, Hong, Krusell, and Rios-Rull \(2012\)](#), [He and Kondor \(forthcoming\)](#) and [Mirza and Stephens \(2016\)](#), among others.⁹ This paper shows how pecuniary externalities may result in inefficient investment when the securitization process is plagued by frictions. Thus, we link the literature on investment with incomplete markets and asset securitization. This allows us to study welfare and examine policies in a well-understood framework.

Our model of securitization is an extension of the framework developed in [Gennaioli, Shleifer, and Vishny \(2013\)](#). In their setting, securitization is socially worthwhile as it can completely remove idiosyncratic risk from financial intermediaries' assets. This permits the financial sector to become more leveraged and invest more.¹⁰ This is inefficient when agents are not rational and cannot assess the risks in securitized assets correctly. In our paper, market incompleteness limits the ability of intermediaries to insure themselves against idiosyncratic asset risk. Furthermore, aggregate investment is excessive due to a coordination failure only when markets are incomplete, which results in inefficient fire-sales even when all agents have rational expectations.

⁹[Krishnamurthy \(2010\)](#) and [Brunnermeier and Oehmke \(2013\)](#) survey the literature on aggregate implications of financial frictions. [Davila \(2015\)](#) provides a discussion of several key papers in this literature, including the impact of various modeling assumptions on welfare analysis. In his terminology, we model a “terms-of-trade” externality.

¹⁰[Diamond \(1984\)](#) first showed risk-pooling by financial intermediaries can increase investment and welfare.

There is a growing literature on securitization, which for the most part focuses on security design and the contractual features arising from asymmetric information cost. For example, [DeMarzo and Duffie \(1999\)](#) and [DeMarzo \(2005\)](#) examine security design problems in static settings whereas [Hartman-Glaser et al. \(2012\)](#) focus on a dynamic problem.¹¹ [Hanson and Sunderam \(2013\)](#) consider a security design problem incorporating endogenous information acquisition by investors. [Shleifer and Vishny \(2010\)](#) develop a novel model of securitization where intermediaries sell assets to maximize fee revenue from intermediation. In contrast, this paper focuses on the general equilibrium impacts of securitized lending and the associated welfare implications. [Gale and Gottardi \(2015\)](#) also examine the impact of pecuniary externalities on ex-ante capital structure of intermediaries. Debt is preferable to equity in their model due to tax advantages, and they identify the impact of fire-sales by bankrupt firms on the ex-ante capital structure. Our paper differs in that we abstract from tax advantages of debt and the possibility of default in equilibrium while also examining the impact of a credit crunch following a fire-sale. [Toda \(2015\)](#) also considers a general equilibrium model of securitization, but abstracts from the pecuniary externalities that are the main focus of this paper.

2 Model

We adapt the model of securitization from [Gennaioli et al. \(2013\)](#). There are three periods; $t = 0, 1, 2$. There are two principal actors; risk-neutral intermediaries (borrowers) and risk-averse investors (savers). We describe each further below.

2.1 Intermediaries

The economy is populated by a measure one of risk-neutral intermediaries, indexed by j , that have access to risky investment projects at both $t = 0$, and $t = 1$ that either succeed

¹¹See [Gorton and Metrick \(2012\)](#) for an excellent summary of the literature and an extended discussion of the relevant issues and outstanding questions.

or fail. Undertaking investment is costly with intermediaries incurring non-pecuniary costs $c(I)$ for investing I units. We assume that $c(\cdot)$ is an increasing and convex function with $c(0) = c'(0) = 0$. We interpret these as the effort costs required to find and maintain quality investments.

Each intermediary has access to risky investment opportunities at $t = 0$ with returns subject to both idiosyncratic and aggregate risk. Investments can either succeed, in which case gross returns per unit of investment are R_0 , or fail and return nothing. Moreover, $t = 0$ investments may succeed early, at $t = 1$, or late, at $t = 2$, while failure is only learned late. The probability of success is identical and independent across intermediaries. The probability of success, $\pi(\omega)$, varies with the aggregate state $\omega \in \Omega \equiv \{g, b, r\}$ realized at $t = 2$. The state g captures a “good” or “growth” state where all intermediaries’ projects succeed, b captures a “bad” state where a number of projects fail, while r captures an even less productive “recession” state such that $\pi(g) = 1 > \pi(b) > \pi(r) = l > 0$. We denote the probability that state ω is realized by $\phi(\omega)$, and define $\phi(g) = p, \phi(b) = (1 - p)q, \phi(r) = (1 - p)(1 - q)$, where $p, q \in (0, 1)$.

At $t = 1$, a fraction $\sigma \in \Sigma \equiv \{h, l\}$ of $t = 0$ projects succeed. We assume σ is publicly observable and provides an informative signal about the realization of the aggregate state ω at $t = 2$. The signal h (high) is observed with probability p and conveys that the aggregate state at $t = 2$ is good (g) with certainty. Alternatively, with probability $1 - p$ the signal l (low) is observed and it conveys that the aggregate state may be either bad (b) or a recession (r).¹² For simplicity, we assume that if the high signal is observed, all investment projects succeed early ($h = 1$), while if the low signal is observed, only a fraction $l < 1$ of projects succeed early. After observing early investment returns, intermediaries form beliefs over late returns. Denote the probability that projects succeed late, conditional on the realization of signal σ by $\pi(\omega|\sigma)$. Then, $\pi(g|h) = 1$ as all projects succeed in the good state, $\pi(r|l) = 0$

¹²Formally, denote by $\phi(\omega|\sigma)$ the probability that the state at $t = 2$ is ω , conditional on the signal σ being observed. We assume $\phi(g|h) = 1, \phi(g|l) = 0, \phi(b|h) = 0, \phi(b|l) = q, \phi(r|h) = 0, \phi(r|l) = 1 - q$. To ensure that signals convey the appropriate information so that $\phi(\omega|\sigma) \neq \phi(\omega)$ for all $\omega \in \Omega, \sigma \in \Sigma$, we impose $h > \pi(b)$ and $l \leq \pi(r)$.

since only l projects succeed in the recession state, and $\pi(b|l) = \pi(b) - \pi(r) = \pi(b) - l > 0$.¹³

Importantly, investments that do not succeed early following the low signal are less likely to ever succeed; i.e., conditional on $\sigma = l$, expected returns of late assets are lower. Thus, while we ignore the possibility of projects failing at time 1, this is consistent with our model and failure of assets at $t = 1$ could be included in the analysis in a straightforward way. We write gross returns on period 0 investments for each intermediary as follows:

$$E_{\omega}(\pi(\omega)) R_0 = \phi(g)\pi(g|h)R_0 + \phi(b)[l + (1-l)\pi(b|l)]R_0 + \phi(r)[l + (1-l)\pi(r|l)]R_0. \quad (1)$$

We assume throughout that investment at $t = 0$ is worthwhile:

ASSUMPTION 1. *For any investment level $I \geq 0$, $E_{\omega}(\pi(\omega)) R_0 I - c(I) > I$.*

Intermediaries also have access to new risky investment opportunities at $t = 1$, which provide returns at $t = 2$. The gross return on one unit of investment is R_1 in the case of success and zero otherwise. The probability of success is q_h when $\sigma = h$, and q_l when $\sigma = l$, where we $q_h > q_l$. This captures the intuitive case in which new opportunities are of higher quality when the high signal is observed. For simplicity, the probability of success is perfectly correlated across intermediaries, and independent of the aggregate state ω at $t = 2$. These assumptions permits us to ignore the possibility of pooling $t = 1$ assets and focus solely on the impact of securitizing $t = 0$ investments. As a further simplification, we let the non-pecuniary costs associated with investment to be zero when $\sigma = h$. We also assume that $t = 1$ projects are always worthwhile.

ASSUMPTION 2. *For any investment level $I \geq 0$, $q_h R_1 I > q_l R_1 I - c(I) > I$.*

Intermediaries' Problem

At $t = 0$, each intermediary invests $I_{0,j}$, and holds reserves $y_{0,j}$, using capital w_{int} and funds raised from investors. To raise funds, intermediaries issue risk-less debt claims D_j at

¹³Note that $\pi(g|l) = \pi(b|h) = \pi(r|l) = 0$ as $\phi(g|l) = \phi(b|h) = \phi(r|h) = 0$.

$t = 0$, that promise a return ρ at $t = 2$. Additional funds for investment can be raised by selling the cash flows associated with $S_{0,j} \leq (1 - a)I_{0,j}$ units of investment, where $a \in [0, 1)$ is the “skin-in-the-game” required by originating intermediaries. As discussed in Section 1, skin-in-the-game requirements can arise from informational frictions in the securitization process and/or regulatory requirements. For simplicity, we take these as given. Intermediaries may also purchase cash flows $T_{0,j}$ from other intermediaries. We interpret $T_{0,j}$ as the cash-flows derived from a pool of all other intermediaries’ assets. Although an intermediary’s own projects have the same expected payoffs as cash-flows purchased from the pool, due to diversification the latter bear no idiosyncratic risk, only aggregate risk. This is important because this diversification allows intermediaries to increase pledgable cash-flows to debt holders when intermediaries have late returns. In other words, the cash-flows from the pool of assets $T_{0,j}$, provide better collateral than the cash-flows from the intermediary’s own investment $I_{0,j}$.

We interpret sales and purchases of cash-flows by intermediaries as securitization. For simplicity we do not model “tranching”, nor do we distinguish between originating intermediaries, sponsoring intermediaries and “special-purpose vehicles” (SPV). We view equity as a junior tranche, which is held by the intermediary, while the senior tranche is sold to investors as (completely) safe debt.¹⁴

The decisions of intermediaries at $t = 1$ consist of investing in new opportunities, purchasing or selling securitized assets from other intermediaries, selling cash flows against their own $t = 0$ investments that have not yet been realized, or holding cash. When $\sigma = h$, all intermediaries are identical as all $t = 0$ investments succeed and are realized early. As a result, there is no motive for trade, and each intermediary makes $I_{1,h,j}$ new investments and holds $y_{1,h,j}$ in cash. When, $\sigma = l$, early intermediaries have more funds available for investment, and thus intermediaries with late returns may have access to relatively profitable investment

¹⁴Ignoring the role of SPVs is not vital since these are typically artificial constructs, operating according to a set of pre-specified rules. A detailed discussion of the process can be found in [Gorton and Metrick \(2012\)](#).

opportunities that cannot be exploited. We denote early types by e and late (not early) types by ne . Early intermediaries invest an amount $I_{1,e,j}$, while late intermediaries invest $I_{1,ne,j}$. Funds may be transferred between intermediaries through the exchange of securitized assets or via the sale of remaining cash-flows on $t = 0$ investment. Importantly, we assume that intermediaries have no other means to generate funds from outsiders. In particular, we rule out lending across intermediaries at $t = 1$. This is done for ease of exposition, but could be included in the analysis below without altering the qualitative results as long as there is a limit on lending between intermediaries, which can be motivated in the same way as the securitization frictions.

We denote early intermediaries' period 1 purchases of securitized assets by $T_{1,e,j}$. Late intermediaries' sales of securitized assets are $-T_{1,ne,j}$, while sales of remaining cash-flows on $t = 0$ assets are $S_{1,ne,j}$. Cash holdings of early and late intermediaries are denoted $y_{1,e,j}$ and $y_{1,ne,j}$ respectively. Formally, intermediary j chooses $I_{0,j}$, $S_{0,j}$, $T_{0,j}$, D_j , $y_{0,j}$, $I_{1,h,j}$, $I_{1,e,j}$, $I_{1,ne,j}$, $T_{1,e,j}$, $T_{1,ne,j}$, $S_{1,ne,j}$, $y_{1,h,j}$, $y_{1,e,j}$, and $y_{1,ne,j}$ to maximize profits Π_j at end of $t = 2$:

$$\Pi_j = p\Pi_{h,j} + (1 - p)[l\Pi_{e,j} + (1 - l)\Pi_{ne,j}] - c(I_{0,j}) - \rho D_j, \quad (2)$$

where

$$\begin{aligned} \Pi_{h,j} &= q_h R_1 I_{1,h,j} + y_{1,h,j}, \\ \Pi_{e,j} &= q_l R_1 I_{1,e,j} - c(I_{1,e,j}) + y_{1,e,j} + q_l(\pi(b) - l)R_0((1 - l)T_{0,j} + T_{1,e,j}), \\ \Pi_{ne,j} &= q_l R_1 I_{1,ne,j} - c(I_{1,ne,j}) + y_{1,ne,j} + q_l(\pi(b) - l)R_0(I_{0,j} - S_{0,j} + (1 - l)T_{0,j} + T_{1,ne,j} - S_{1,ne,j}). \end{aligned}$$

The first term in (2), $\Pi_{h,j}$, is expected profit at period 2 following a high signal at $t = 1$. In this case, all $t = 0$ projects succeed early and the proceeds are either re-invested in new opportunities, with gross returns $q_h R_1 I_{1,h,j}$, or held as reserves, $y_{1,h,j}$. We assume that there are no additional costs associated with making new investments. Also, as intermediaries are identical in this case, and each has sufficient funds to repay investors, there is no motive

for trade. The second term in (2) is expected profit at period 2 following a low signal at $t = 1$. Expected profits are a weighted sum of early and late types' profits, $\Pi_{e,j}$ and $\Pi_{ne,j}$ respectively. Profits for early types consist of returns on new investment, $q_l R_1 I_{1,e,j} - c(I_{1,e,j})$, reserves carried into period 2, $y_{1,e,j}$, and late returns on securitized assets purchased either at $t = 0$ or $t = 1$, $q_l(\pi(b) - l)R_0((1 - l)T_{0,j} + T_{1,e,j})$. Similarly, profits for late types consists of investment returns, $q_l R_1 I_{1,ne,j} - c(I_{1,ne,j})$, reserves $y_{1,ne,j}$, and late returns on assets not sold, $q_l(\pi(b) - l)R_0(I_{0,j} - S_{0,j} + (1 - l)T_{0,j} + T_{1,ne,j} - S_{1,ne,j})$. Finally, the last two terms in (2) capture the costs of investment in the initial period, $c(I_{0,j})$, and debt repayment, ρD_j .

Intermediaries maximize (2) subject to the following set of constraints:

$$(\lambda_{0,j}) \quad I_{0,j} + p_0(T_{0,j} - S_{0,j}) + y_{0,j} \leq w_{int} + D_j, \quad (3)$$

$$(\mu_{0,j}) \quad S_{0,j} \leq (1 - a)I_{0,j}, \quad (4)$$

$$(\lambda_{1,h,j}) \quad I_{1,h,j} + y_{1,h,j} \leq R_0(I_{0,j} + T_{0,j} - S_{0,j}) + y_{0,j}, \quad (5)$$

$$(\lambda_{1,e,j}) \quad I_{1,e,j} + p_1 T_{1,e,j} + y_{1,e,j} \leq R_0(I_{0,j} - S_{0,j}) + l R_0 T_{0,j} + y_{0,j}, \quad (6)$$

$$(\lambda_{1,ne,j}) \quad I_{1,ne,j} + p_1(T_{1,ne,j} - S_{1,ne,j}) + y_{1,ne,j} \leq l R_0 T_{0,j} + y_{0,j}, \quad (7)$$

$$(\mu_{1,S,j}) \quad S_{0,j} + S_{1,ne,j} \leq (1 - a)I_{0,j}, \quad (8)$$

$$(\mu_{1,T,j}) \quad 0 \leq T_{1,ne,j} + (1 - l)T_{0,j}, \quad (9)$$

$$(\eta_{1,h,j}) \quad \rho D_j \leq y_{1,h,j}, \quad (10)$$

$$(\eta_{1,e,j}) \quad \rho D_j \leq y_{1,e,j}, \quad (11)$$

$$(\eta_{1,ne,j}) \quad \rho D_j \leq y_{1,ne,j}. \quad (12)$$

Inequality (3) is the budget constraint at $t = 0$, which requires investment costs, net purchases and reserves be no greater than equity and debt. The second constraint is the skin-in-the-game requirement. Expression (5) is the budget constraint when $\sigma = h$. Inequalities (6) and (7) are the budget constraints of the early and late intermediaries at $t = 1$ when the low signal is realized. Early intermediaries have $R_0(I_{0,j} - S_{0,j})$ more funds than late

types, as their projects succeed early. Early intermediaries can then use the returns from their individual investments along with securitized assets to purchase assets from late ones or invest in new opportunities and reserves. Late intermediaries use returns from securitized assets, plus funds raised from asset sales, to finance new investment and reserves. Constraints (8) and (9) ensure that individual and securitized asset sales are feasible and satisfy the skin-in-the-game requirement ($T_{1,ne,j} < 0$ represent sales). The final set of constraints are the intermediaries' collateral constraints that ensure debt is always repaid.¹⁵ The solution to this problem is characterized in Appendix A, where the Lagrange multipliers associated with each constraint are given in brackets above.

2.2 Investors

We assume there is a measure one of identical (infinitely) risk-averse investors, each endowed with a large positive wealth w_i . Investors value consumption only at $t = 0$ and $t = 2$ with preferences given by

$$U_i = C_{0,i} + \beta E_\omega \left[\min_\omega \{C_{2,\omega,i}\} \right], \quad (13)$$

where $C_{0,i}$ is consumption at $t = 0$, $C_{2,\omega,i}$ is consumption at $t = 2$ in state ω , and $\beta \in (0, 1)$ is a discount factor. While these preferences are extreme, there is evidence that many investors in securitized products, such as mutual/pension funds, require a high degree of safety.¹⁶ At $t = 0$, investors can either consume their wealth, purchase risk-less debt from intermediaries that pays a gross interest rate ρ at $t = 2$, or buy risky assets issued by intermediaries. At $t = 1$, investors can additionally purchase securities issued by intermediaries. Investor i 's

¹⁵We assume that intermediaries and investors can commit to long term contracts. Relaxing this assumption in our environment has no effect on our results, however this represents an potentially interesting extension in a model with more general investor preferences. For example, we could interpret our contracts as short-term, which may be rolled over at $t = 1$. This is done in Stein (2012), although he ignores the potential for renegotiation at time 1.

¹⁶For example, see Bernanke, Bertaut, Demarco, and Kamin (2011), Stein (2012), and Gennaioli et al. (2013) for a discussion on the desire for safety on the part of institutional investors.

budget constraints at $t = 0, 1, 2$ are thus

$$C_{0,i} + D_i + p_0 T_{0,i} + y_{0,i} \leq w_i, \quad (14)$$

$$p_1 T_{1,i} + y_{1,i} \leq y_{0,i} + l R_0 T_{0,i}, \quad (15)$$

$$C_{2,\omega,i} \leq y_{1,i} + \rho D_i + (1-l)\omega R_0 T_{0,i} + (\pi(\omega) - l) R_0 T_{1,i}, \quad (16)$$

where D_i is the amount of risk-less debt purchased, $y_{0,i}$ and $y_{1,i}$ are the amounts of cash held at $t = 0, 1$, and $T_{0,i}$ and $T_{1,i}$ are the quantities of intermediary assets purchased at $t = 0, 1$.

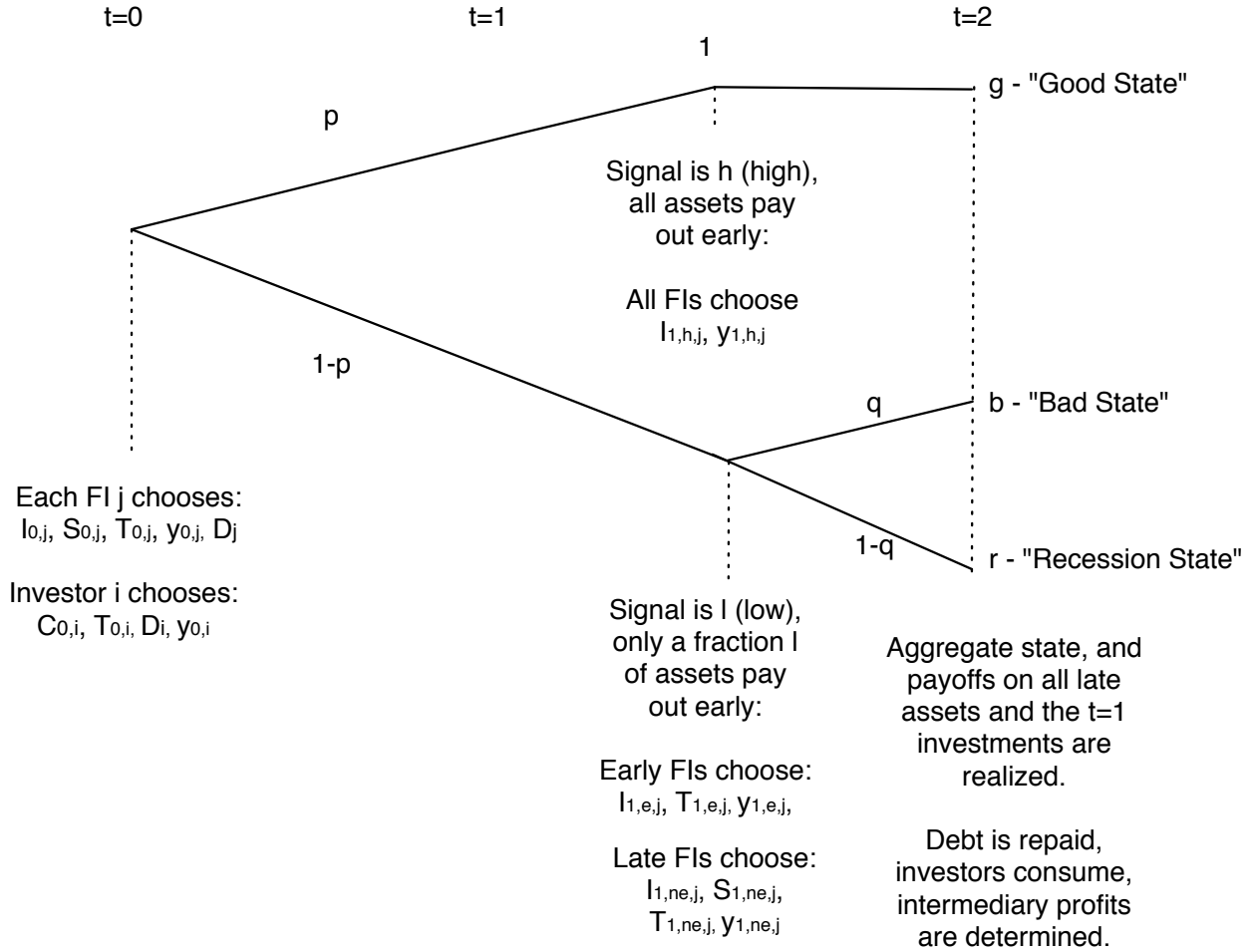


Figure 1: Timing.

2.3 Equilibrium Definition

The intermediary problem is to choose investment, reserves, trade and debt levels to maximize expected profits subject to budget, collateral, sales, and investors' participation constraints. The investor problem is to choose how much debt and securities issued by intermediaries to purchase (if any), and savings to maximize expected utility of consumption subject to budget constraints. The price of debt, ρ , and the prices of securities p_0, p_1 , are taken as given by intermediaries and investors. Our concept of equilibrium is characterized in the following definition.

DEFINITION 1. *A symmetric competitive equilibrium consists of prices ρ, p_0, p_1 , and choices of investment $I_{0,j}, I_{1,h,j}, I_{1,e,j}, I_{1,ne,j}$, reserves $y_{0,j}, y_{1,h}, y_{1,e}, y_{1,ne}$, trade $T_{0,j}, S_{0,j}, T_{1,ne}, S_{1,ne}$, and debt D_j by each intermediary j , and choices of debt D_i , securities purchases $T_{0,i}, T_{1,i}$, and savings $y_{0,i}, y_{1,i}$ for each investor i , such that given prices:*

1. *Investors maximize expected utility (13) s.t. (14)-(16),*
2. *Intermediaries maximize expected profits (2) s.t. (3)-(12),*
3. *Markets clear:*

$$\text{Market for debt at } t = 0 : \int D_i di = \int D_j dj, \quad (17)$$

$$\text{Market for assets at } t = 0 : \int T_{0,j} dj + \int T_{0,i} di = \int S_{0,j} dj, \quad (18)$$

$$\text{Market for assets at } t = 1 : \int_{\{e\}} T_{1,e,j} dj + \int T_{1,i} di = \int_{\{ne\}} S_{1,ne,j} dj, \quad (19)$$

where $\{e\}$ and $\{ne\}$ are the set of early and late types.

3 Equilibrium

In this section we characterize the competitive market equilibrium. We first solve for the optimal decisions of investors and intermediaries given asset prices and the interest rate on

debt. We then show how market clearing determines equilibrium prices by aggregating these decisions.

3.1 Optimal Decisions of Investors

Investor behavior is straightforward in that they either consume their endowments, or purchase risk-less debt. Throughout the remainder of the paper, we denote equilibrium values with an asterisk.

$$\textbf{LEMMA 1. } T_{0,i}^* = y_{0,i}^* = T_{1,i}^* = y_{1,i}^* = 0 \text{ and } D_i = \begin{cases} 0, & \text{if } \rho < \beta^{-1} \\ [0, w_i], & \text{if } \rho = \beta^{-1} \\ w_i, & \text{if } \rho > \beta^{-1}. \end{cases}$$

Proof. See Appendix B. □

Investors only value assets at the lowest realization and thus are priced out of the market for assets by intermediaries. Moreover, investor preferences ensure that all debt is risk-free and hence their break-even condition on funds lent to intermediaries, $\rho \geq \beta^{-1}$, places a lower bound on the equilibrium interest rate.

3.2 Optimal Decisions of Intermediaries

The following assumption ensures that intermediaries borrow a positive amount, so that taking on debt to expand investment is worthwhile.

$$\textbf{ASSUMPTION 3. } E_{\omega}(\pi(\omega)) R_0 - c'(w_{int}) > \beta^{-1} > E_{\omega}(\pi(\omega)) R_0 - c'(w_{int} + w_i).$$

This assumption implies that the marginal return on investment using borrowed funds is eventually negative despite the investments having positive NPV due to Assumption 1. This is illustrated in Figure 2. The implications of this are characterized formally in Lemma 2, which states that they hold no reserves at $t = 0$, and hold exactly the quantity of reserves at $t = 1$ as needed to service debt at $t = 2$.

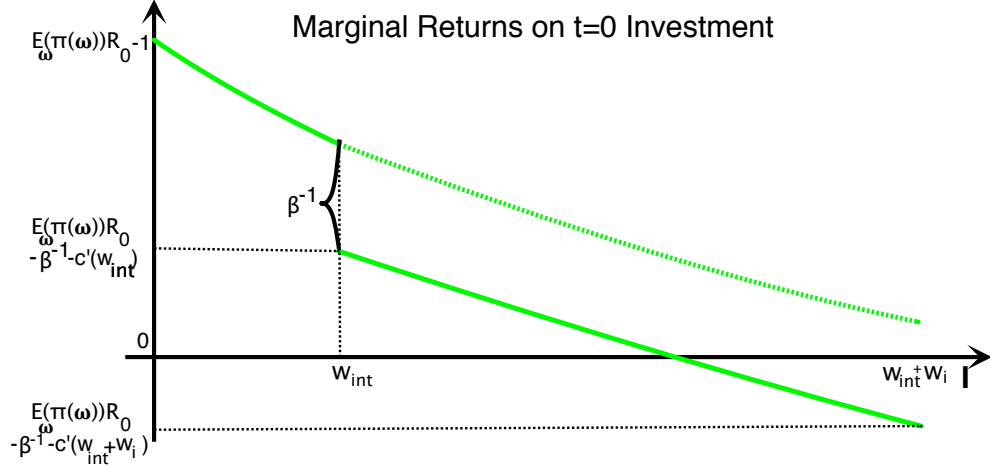


Figure 2: Illustration of Assumption 3.

LEMMA 2. *Demand for debt is downward sloping, $\partial D_j^*/\partial \rho < 0$, $D_j > 0$ only if $\rho \leq E_w(\pi(\omega)) R_0 - c'(w_{int})$, $y_{0,j}^* = 0$, $y_{1,h,j}^* = y_{1,e,j}^* = y_{1,ne,j}^* = \rho D_j^*$.*

Proof. See Appendix B. □

Demand for debt is downward sloping due to the diminishing returns on investment at $t = 0$. Given that intermediaries have a strictly positive amount of equity, a sufficiently small level of debt can always be repaid so that $D_j > 0$ in equilibrium. This requires the interest rate be below the marginal return on the first unit of debt, or $E_w(\pi(\omega)) R_0 - c'(w_{int})$. Moreover, it is never optimal for intermediaries to finance reserve holdings at $t = 0$ via debt. This is because for every unit of debt raised at $t = 0$, intermediaries must generate $\rho - 1 \geq \beta^{-1} - 1 > 0$ at $t = 1$ to service this additional unit of debt. To understand reserve holdings at period 1, note that neither returns from new investments at $t = 1$ nor late returns on $t = 0$ can be pledged to repay investors at $t = 2$. This is because infinitely risk-averse investors value these pledges at the lowest possible return which is zero. Hence, intermediaries hold reserves equal to ρD_j for all $\sigma \in \Sigma$. Note that intermediaries always have sufficient funds to build up the required reserves, otherwise no funds would be lent in equilibrium given investor preferences.

We now focus on the optimal investment and trading decisions of intermediaries at period 1, taking as given period 0 decisions. When $\sigma = h$, intermediaries are identical at $t = 1$ as

all $t = 0$ investments succeed early, and hence there is no trade. In this case, intermediaries simply set aside the required reserves and invest the remainder in new opportunities since these are always worthwhile, from Assumption 2. Hence, $I_{1,h,j} = R_0 I_{0,j} - \rho D_j$. When $\sigma = l$, intermediaries differ at period 1. A fraction l receive the full return on the fraction of $t = 0$ investments that were not securitized. The remaining fraction $(1 - l)$ do not receive any early returns on their own investments. Due to securitization, all intermediaries also receive a fraction of the early returns from other intermediaries' projects. For a given p_1 , early types can use their funds to either invest in new opportunities or purchase assets from late types. The amount of new investment, $I_{1,e,j}$, equates the marginal return to investment with the marginal return on purchasing assets. The former is simply $q_l R_1 - c'(I_{1,e,j})$ while the latter is $q_l(\pi(b) - l)R_0/p_1$ where $q_l(\pi(b) - l)R_0$ is the net present value on $t = 0$ investments, conditional on the low signal at $t = 1$.

LEMMA 3. *Demand for assets is downward sloping, $\partial T_{1,e,j}/\partial p_1 < 0$. Let $\underline{p}_1 = (\pi(b) - l)R_0/R_1$, early types $t = 1$ investment and asset purchases, $I_{1,e,j}$ and $T_{1,e,j}$, are given by the following.*

$$\begin{aligned}
\text{For } p_1 < \underline{p}_1: \quad & I_{1,e,j} = 0 \\
& T_{1,e,j} = \frac{aR_0 I_{0,j} + lR_0 T_{0,j} - \rho D_j}{p_1}. \\
\text{For } p_1 \geq \underline{p}_1: \quad & q_l R_1 - c'(I_{1,e,j}) = \frac{q_l(\pi(b) - l)R_0}{p_1} \Rightarrow \frac{\partial I_{1,e,j}}{\partial p_1} > 0, \\
& T_{1,e,j} = \frac{aR_0 I_{0,j} + lR_0 T_{0,j} - \rho D_j - I_{1,e,j}}{p_1}.
\end{aligned}$$

Proof. See Appendix B. □

As the price p_1 increases, the return on purchasing assets is lower and therefore more investment is undertaken and fewer assets are purchased by early types. Analogously, for lower values of p_1 , early types purchase more assets, and invest less.

The investment and sales decisions by late types involve a similar trade-off. By selling

$t = 0$ assets late types forgo the returns, but can increase new investment and/or generate reserves required to service debt. Sales consist of securitized assets on hand, $-T_{1,ne,j}$ as well as any of their own investments which were not sold at $t = 0$, $S_{1,ne}$. Late types may be constrained if they run out of $t = 0$ assets to sell, in which case the multiplier on the sales constraint will bind, i.e., $\mu_{1,T,j} > 0$.

LEMMA 4. *Let $\underline{p}_1 = (\pi(b) - l)R_0/R_1$, late types $t = 1$ investment and asset purchases, $I_{1,ne,j}$, and $S_{1,ne,j} - T_{1,ne,j}$ are characterized as follows.*

$$\begin{aligned}
\text{For } p_1 < \underline{p}_1 : \quad & I_{1,ne,j} = 0 \\
& S_{1,ne,j} - T_{1,ne,j} = \frac{\rho D_j - l R_0 T_{0,j} - y_{0,j}}{p_1} \\
\text{For } p_1 \geq \underline{p}_1 : \quad & q_l R_1 - c'(I_{1,ne,j}) = \frac{q_l(\pi(b) - l)R_0}{p_1} + \frac{\mu_{1,T,j}}{(1-p)(1-l)}, \\
& S_{1,ne,j} - T_{1,ne,j} = \min \left[\frac{I_{1,ne,j} + \rho D_j - l R_0 T_{0,j}}{p_1}, (1-l)T_{0,j} + (1-a)I_{0,j} - S_{0,j} \right].
\end{aligned}$$

Proof. See Appendix B. □

As can be seen from Lemmas 3 and 4, if $\mu_{1,T,j} > 0$, equilibrium investment levels will differ across types at $t = 1$. As a result, when late types are constrained at $t = 1$, intermediaries always find it optimal to securitize as much as possible at $t = 0$. This is formalized in the following result.

LEMMA 5. $\mu_{1,T,j} > 0 \iff I_{1,e,j}^* > I_{1,ne,j}^* \iff \mu_{0,j} > 0 \text{ and } \mu_{1,S,j} = 0$.

Proof. See Appendix B. □

The key friction built in to our framework is that late intermediaries may only trade assets at $t = 1$ to generate funds for investment. If late intermediaries cannot raise sufficient funds, $I_{1,e,j}^* > I_{1,ne,j}^*$. Furthermore, when $\mu_{1,T,j} > 0$, securitized assets are worth more than individual investment holdings, since they provide relatively more resources to late types who value them more. As result, being constrained at $t = 1$ means that intermediaries

will securitize to the extent possible at $t = 0$, so that $\mu_{0,j} > 0$. For simplicity, we shall assume throughout that intermediaries always securitize $t = 0$ assets to the extent possible, regardless of the value of $\mu_{1,T,j}$. Hence, $S_{0,j}^* = T_{0,j}^* = (1 - a)I_{0,j}^*$.¹⁷ We now consider the investment decision at period 0.

LEMMA 6. $I_{0,j}$ is characterized by

$$E_\omega(\pi(\omega))R_0 - \rho + p(R_0 - \rho)(q_h R_1 - 1) + (1 - p)l((a + (1 - a)l)R_0 - \rho)(q_l R_1 - 1 - c'(I_{1,e})) \\ + (1 - p)(1 - l)((1 - a)lR_0 - \rho)(q_l R_1 - 1 - c'(I_{1,ne})) + \mu_{1,T,j}(1 - a)(1 - l) = c'(I_{0,j}). \quad (20)$$

Proof. See Appendix B. □

The marginal return to a unit of investment at $t = 0$, given that $D_j^* > 0$ is $E_\omega(\pi(\omega))R_0 - \rho$ plus the marginal returns from re-investing early returns at $t = 1$. When the signal is high, each additional unit of $I_{0,j}$ generates $R_0 - \rho$ units of resources at $t = 1$ that can be reinvested for a gross expected return $q_h R_1$. Similarly, when the signal is low, another unit of $I_{0,j}$ generates $(a + (1 - a)l)R_0 - \rho$ units of resources for the early types and $(1 - a)lR_0 - \rho$ units of resources for the late types. These can be reinvested at gross returns of $q_l R_1 - c'(I_{1,e})$ and $q_l R_1 - c'(I_{1,ne})$. If the sales constraint binds, increasing $I_{0,j}$ provides an additional benefit: it raises by $(1 - a)(1 - l)$ units the quantity of assets late types can sell (although this can increase total liabilities). The optimal choice of $I_{0,j}$ then equates the marginal benefit of investment with the corresponding marginal cost, $c'(I_{0,j})$.¹⁸

¹⁷Lemma 5 shows that this is consistent with optimal behavior when constrained. However, this is somewhat arbitrary for the unconstrained case, since the optimal choices of $S_{0,j}$ and $T_{0,j}$ are not well defined. This is not significant for our results however, since our focus is on equilibria in which intermediaries are constrained (the reasons for which are made clear in Section 4).

¹⁸If intermediaries incur losses at $t = 1$, these will be borne by their equity. As a result, this may place an upper bound on the level of initial investment. However, we assume that the marginal costs associated with investment at $t = 0$ are sufficiently high so that (20) always holds (otherwise there is no investment at $t = 1$). A sufficient condition for an interior solution for $I_{0,j}$ is

$$p(R_0 - \rho)(q_h R_1 - 1) + (1 - p)l(R_0 - \rho)(q_l R_1 - 1) + E_\omega(\pi(\omega))R_0 - \rho < c' \left(\frac{-\rho w_{int}}{lR_0 - \beta^{-1}} \right).$$

3.3 Market Clearing

From the optimal choices of investors and intermediaries, we can infer that ρ must satisfy the following bounds $E_\omega(\pi(\omega)) R_0 - c'(w_{int}) \geq \rho \geq \beta^{-1}$. In fact, given that demand for debt is downward sloping (Lemma 1), and supply is perfectly elastic at a price of β^{-1} (Lemma 2), together with Assumption 3 requires that in equilibrium $\rho = \beta^{-1}$. This is illustrated graphically in Figure 3.

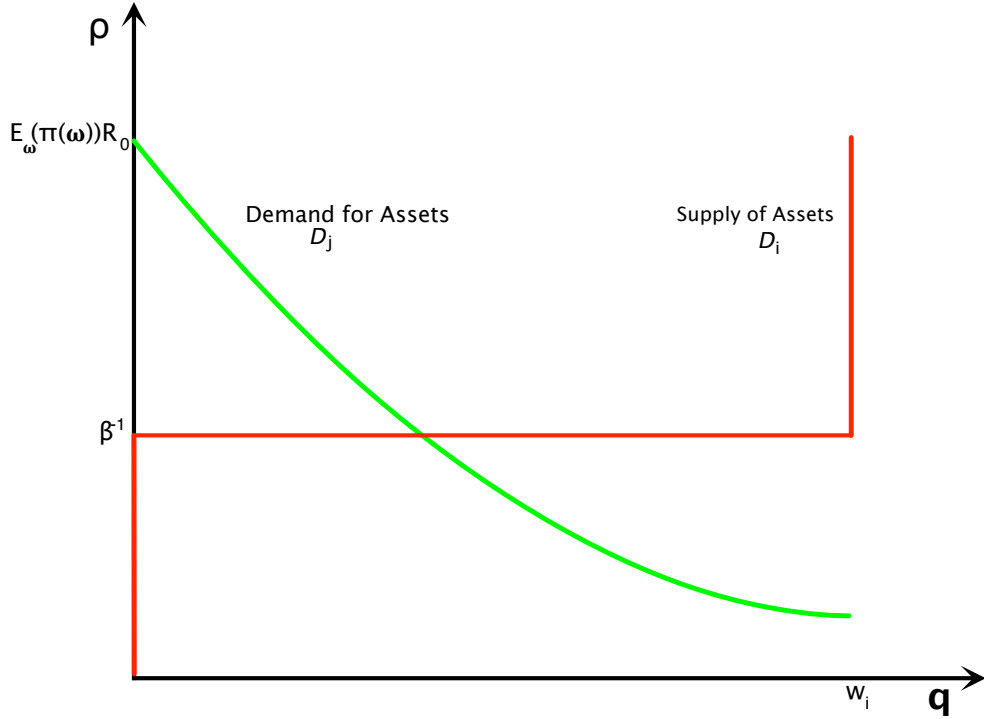


Figure 3: Market for debt.

Consider the $t = 0$ market for securitized assets. It is shown in Lemma 5 that when constrained, $S_{0,j} = T_{0,j} = (1 - a)I_{0,j}$. When unconstrained, intermediaries are indifferent over their choices of $T_{0,j}$ and $S_{0,j}$, so we simply assume they securitize all assets. Regardless of the choices of $T_{0,j}$ and $S_{0,j}$, any candidate equilibrium price \tilde{p}_0 must clear the market, and thus $S_{0,j}(\tilde{p}_0) - T_{0,j}(\tilde{p}_0) = 0$. Inspecting the intermediaries' problem, it is clear that \tilde{p}_0 has no effect on the budget, since all agents are identical and net purchases are zero. Thus optimal choices are determined by the first order conditions from the intermediaries' problem at a

given p_1 , which are provided in Appendix A. The $t = 0$ price that clears the market satisfies

$$p_0 = \frac{a\mu_{0,j} + c'(I_{0,j}^*)}{\lambda_{0,j}} + 1. \quad (21)$$

In an unconstrained equilibrium, $p_0 = c'(I_{0,j}^*)/\lambda_{0,j} + 1$ is simply the marginal cost of time 0 investment. When constrained, p_0 is strictly larger than the unconstrained case, which reflects the fact that securitized assets are relatively valuable in this type of equilibrium as they provide more resources to late types in the low state of the world.

We now consider the determination of p_1 . Investment and securitization at $t = 0$ influence p_1 and late types may or may not be constrained, illustrated in Figures 4 and 5. From the optimal choices of intermediaries described in Lemmas 3 and 4, we can infer the following bounds on p_1 : $q_l(\pi(b) - l)R_0 \geq p_1 \geq q_l(\pi(b) - l)R_0/q_l R_1$ when $t = 1$ investment is positive. To understand these bounds, note that if p_1 were to exceed the conditional return on assets, early types would not be willing to purchase them, since they can always invest in new projects that earn positive profit. Thus, at the equilibrium, assets will only trade at fire-sale prices (i.e. below NPV). Similarly, if p_1 is below $q_l(\pi(b) - l)R_0/q_l R_1$, early types do not make any new investments as buying up cheap assets is more profitable. The following proposition ensures the existence of a unique equilibrium.

PROPOSITION 1. *There exists a unique symmetric competitive equilibrium with positive investment at both periods.*

Proof. See Appendix B. □

To more fully characterize the nature of the equilibrium we need to further characterize the supply curve at $t = 1$, which depends on the relative strength of the two motives for selling; namely to generate collateral and fund new investment. When the collateral motive dominates, the supply curve is downward sloping. When the investment motive is sufficiently strong, the supply curve is upward sloping, which is only possible for prices

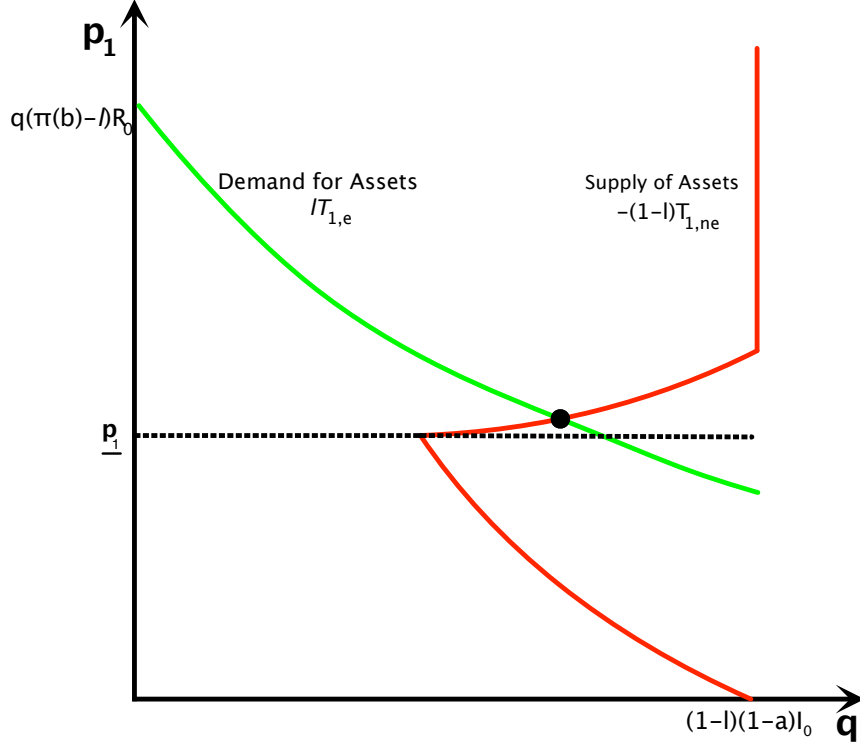


Figure 4: Unconstrained equilibrium.

above $(\pi(b) - l)R_0/R_1$. Sufficient conditions for this case to obtain are given in the following assumption.

ASSUMPTION 4.

I. $c''' > 0$, and $c''(w_i(lR_0 - \beta^{-1}) + w_{int}lR_0) \geq \frac{1}{q_l(\pi(b)-l)R_0(1-a)(w_i+w_{int})}$, or

II. $c''' < 0$ and $\frac{1}{c''(0)} \geq q_l(\pi(b) - l)R_0(1 - a)(1 - l)(w + w_{int})$.

COROLLARY 1.1. *Given Assumption 4, if $q_l(\pi(b) - l) \leq \frac{al}{(1-a)(1-l)}$, then a unique constrained equilibrium exists where $\mu_{1,T,j} > 0$.*

Proof. See Appendix B. □

We focus below on equilibria in which $\mu_{1,T,j} > 0$. Assumption 4 ensures that the supply curve at $t = 1$ is upward sloping, which is required for constrained equilibria. Corollary 1.1 shows that parameters exist which support this type of situation. The sufficient condition

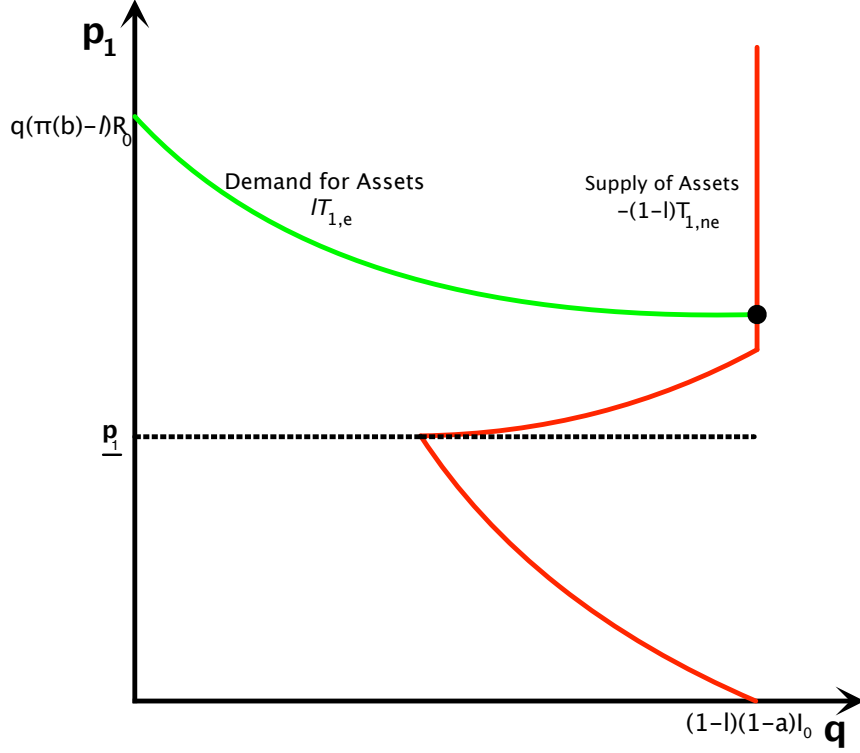


Figure 5: Constrained equilibrium.

in Corollary 1.1 suggests that constrained equilibria arise when a and l are relatively large and/or the returns on $t = 0$ assets, conditional on $\sigma = l$, are small. Respectively, this means there is more heterogeneity at $t = 1$, more cash in the market (demand) and the conditional value of the assets is smaller so that, ceteris paribus, more assets need to be sold to generate cash.

4 Welfare

In this section, we focus on the efficiency of allocations at the competitive equilibrium. We begin by characterizing the first-best allocation and then consider the welfare implications of market incompleteness and securitization for competitive equilibria.

4.1 First-Best

In the market equilibrium, investor i 's utility is simply w_i , since $\rho = 1/\beta$. It is sufficient for our purposes to focus on the point of the first-best frontier associated with this level of investor utility. Thus, we consider a social planner that collectively maximizes intermediary returns, subject to $\rho = 1/\beta$. Decisions are made ex-ante and all intermediaries are equivalent from a welfare perspective, thus we ignore subscripts and allow the planner to directly choose the aggregate quantities $I_0, D, y_0, I_{1,h}, I_{1,e}, I_{1,ne}, y_{1,h}, y_{1,l}$ to maximize:

$$\begin{aligned} \Pi^P = & p [q_h R_1 I_{1,h} + y_{1,h}] + (1 - p) [l(q_l R_1 I_{1,e} - c(I_{1,e})) \\ & + (1 - l)(q_l R_1 I_{1,ne} - c(I_{1,ne})) + y_{1,l}] - c(I_0) - \rho D, \end{aligned} \quad (22)$$

subject to the following budget and collateral constraints:

$$(\lambda_0) \quad I_0 + y_0 \leq w_{int} + D, \quad (23)$$

$$(\lambda_{1,h}) \quad I_{1,h} + y_{1,h} \leq R_0 I_0 + y_0, \quad (24)$$

$$(\lambda_{1,l}) \quad l I_{1,e} + (1 - l) I_{1,ne} + y_{1,l} \leq l R_0 I_0 + y_0, \quad (25)$$

$$(\eta_{1,h}) \quad \rho D \leq y_{1,h}, \quad (26)$$

$$(\eta_{1,l}) \quad \rho D \leq y_{1,l}. \quad (27)$$

We refer to the solution of this problem as the first-best. The planner maximizes all intermediary profits simultaneously, subject to a single budget constraint in each state. As shown in proof of Proposition 2, the first-best requires equal investments for early and late intermediary types at $t = 1$, which is not necessarily the case in the private market equilibrium. When markets are incomplete, this can result in an inefficiency that does not occur when markets are complete, as outlined in the following proposition.

PROPOSITION 2. *When intermediaries have access at $t = 0$ to a complete set of one-period securities that are contingent on the signal, as well as individual type at $t = 1$, the*

competitive equilibrium allocation is first-best.

Proof. See Appendix B. □

Securitized assets are unnecessary when intermediaries can trade one-period contingent securities at time 0. When markets are not complete, i.e., individual type is not contractible, trading in securitized assets at $t = 0$ may be useful since this can move resources at $t = 1$ from intermediaries whose assets have paid out to those whose have not. This is especially valuable when intermediaries face binding constraints at time 1, since those whose assets have not paid out at $t = 1$ place higher value on cash. The following result shows that when there are no frictions in the process, securitization completes markets in this environment.

PROPOSITION 3. *When there are no contingent securities at $t = 0$, but securitization is frictionless, so that $a = 0$, the private market equilibrium allocation is first-best.*

Proof. See Appendix B. □

When there are no frictions in the securitization process, contingent securities are not necessary to achieve the first-best. This is because intermediaries that are constrained at $t = 1$ find it worthwhile to securitize their assets at $t = 0$. When $a = 0$, this means that the constrained equilibrium is ruled out since all assets are securitized and there is no heterogeneity (and thus no trade) ex-post.¹⁹ It is important to stress at this point that we rule out borrowing between intermediaries at $t = 1$. If there were frictionless borrowing at $t = 1$, then this also delivers the first-best without the need for securitization or contingent contracts.

¹⁹While we assume $T_{0,j} = S_{0,j} = (1 - a)I_{0,j}$, intermediaries may not securitize all assets if they do not anticipate being constrained, and in this case there may be differences across types at $t = 1$. However, we show below that this is not inefficient since this results only in cash transfers across risk-neutral parties and does not affect aggregate investment. Note that if intermediaries were risk-averse, they would always securitize the full amount since this has insurance value which is irrelevant in our setting.

4.2 Second-Best and Inefficient Investment

We now consider the case where markets are incomplete and there are frictions associated with the securitization process. When $a > 0$ and there are no alternative funds available at $t = 1$, late intermediaries must secure funding through asset sales on the spot market. In this case, there is trade at $t = 1$ and atomistic intermediaries fail to endogeneize the price effects of their time 0 decisions. This can result in constrained inefficiency, in the sense that a planner can engineer Pareto improvements even when subject to the same restrictions as the private economy, unlike the Planner described in Section 4.1.

To show that the competitive equilibrium can be constrained inefficient, consider a perturbation of aggregate investment $dI_0 = \int_j dI_{0,j}$, such that $dI_{0,j}$ is equal for all j . The effect of this perturbation on intermediary j 's profits at the competitive equilibrium is:

$$\frac{d\Pi_j^*}{dI_0} = \lambda_{1,e,j}^* \left(-\frac{dp_1^*}{dI_0} T_{1,e,j}^* \right) + \lambda_{1,ne,j}^* \left(-\frac{dp_1^*}{dI_0} (T_{1,ne,j}^* - S_{1,ne,j}^*) \right). \quad (28)$$

The direct impacts of the change are zero at the equilibrium allocation, which satisfies the individual first-order conditions. What remains are the price effects that arise from a change in aggregate investment, something not considered by individual decision makers. Note that such a perturbation generally affects both prices p_0 and p_1 , however changes in p_0 have no impact on time 0 intermediaries at the equilibrium, since each has net securitized assets purchases of zero. Importantly, if $d\Pi_j^*/dI_0 \neq 0$, the equilibrium is constrained inefficient. To see this, consider a constrained planning problem in which the planner is forced to make use of the same instruments as private intermediaries. Thus, the planner solves the private intermediary problem, except that a planner considers the implications of aggregate investment and thus endogeneizes the price effects described in (28). Therefore, (28) is precisely the difference between the individuals' first order condition on $t = 0$ investment and second-best planner's problem. The following proposition summarizes the inefficiency.

PROPOSITION 4. *When $a > 0$, the competitive market equilibrium is constrained ineffi-*

cient whenever $\mu_{1,T,j} > 0$. Parameters exist which support either over or under investment.

- $lR_0 - \frac{1}{\beta} > 0$ is sufficient for under-investment.
- $\frac{1}{\beta} - lR_0 > a(1-l)R_0$ is sufficient for over-investment.

Proof. See Appendix B. □

When unconstrained, the marginal return to $t = 1$ investment is equalized across intermediaries and thus $t = 0$ decisions reflect the full social cost and benefit of investment. More specifically, individual intermediaries do not internalize the impacts of $I_{0,j}$ on aggregate cash in the market and thus prices, which they take as given. The price impacts of individual decisions result in inefficiency because they influence aggregate investment and do not merely represent cash redistributions at $t = 1$ (which have no welfare implications). When $lR_0 - 1/\beta > 0$, $t = 0$ investment generates additional aggregate cash in the market at $t = 1$, which results in a higher asset price and welfare improvements. If $lR_0 - 1/\beta$ is sufficiently negative on the other hand, this has the opposite effect and results in over-investment.

4.3 Policy and over-investment

We now consider the use of two real-world regulatory instruments that are relevant to the inefficiency in our model. In this section we focus solely on the over-investment case. The following corollary of Proposition 4 considers the direct impact of restricting investment.

COROLLARY 4.1. *Restrictions on leverage increase welfare when the competitive equilibrium is characterized by over-investment.*

A reduction in leverage in our framework is equivalent to a reduction in initial investment and the result is obtained directly from Proposition 4. This increases welfare due to the increase in p_1 , which effectively transfers resources from early to late types. While this is the most direct approach, this policy requires that a regulator have the information and power to impose such a restriction. An alternative possibility, which may be relevant for

the reasons discussion in Section 1, is to reduce the collateral value of investment by further tightening the skin-in-the-game constraint. In our model, this can be analyzed with the following comparative static result.

PROPOSITION 5. *In the over-investment case, welfare decreases when skin-in-the-game requirements are tightened, $d\Pi_j^*/da < 0$.*

Proof. See Appendix B. □

Consider,

$$\frac{d\Pi_j^*}{da} = -\mu_{0,j}^* I_{0,j}^* - \frac{dp_1^*}{da} [\lambda_{1,e,j}^* T_{1,e,j}^* + \lambda_{1,ne,j}^* T_{1,ne,j}^*]. \quad (29)$$

First note that this derivative is zero in the unconstrained case. This is because production at early and late types is identical when unconstrained, and a change in a amounts to a redistribution of cash between risk-neutral individuals in period 1. When constrained, the first term in the expression above is the direct effect, which is always negative and captures the fact that increasing skin-in-the-game requirements leads to lower investment at $t = 0$ as it restricts the ability of intermediaries to generate collateral. The second term is the indirect effect, which captures the change in the equilibrium price due to changes in a through the effect on aggregate investment at $t = 0$. Increasing a will tend to reduce investment at $t = 0$ which will reduce the severity of the fire-sales at $t = 1$. As a result, the price of assets at $t = 1$ are pushed up. If the price effect dominates the direct effect, it seems plausible that a regulator could achieve a Pareto improvement by tightening the skin-in-the-game requirements (at least for some parameterizations). The proof of Proposition 5 shows that the price effect is never sufficient to offset the direct reduction in welfare from the restriction. The reason for this is that a price increase raises collateral value and thus investment, defeating the purpose of the policy. Thus, while securitization leads to more investment/leverage, which is already excessive from a second-best perspective, this is always welfare improving. The impact on the equilibrium price p_1 due to an increase in a is illustrated in Figure 6.

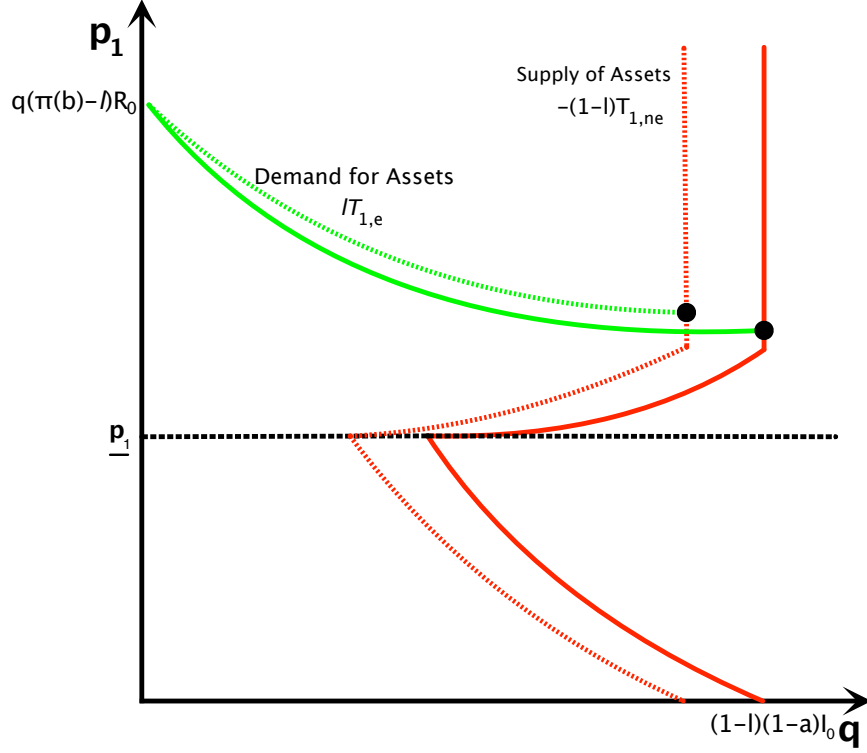


Figure 6: Impact on $t = 1$ equilibrium of tightening skin-in-the-game requirements in the case of over-investment.

5 Conclusion

In the absence of market frictions, securitization provides valuable risk-sharing in the financial sector when there are incomplete markets. When intermediaries are forced to hold some of the idiosyncratic risk associated with their investments however, a pecuniary externality can generate inefficient investment ex-ante and excessive fire-sales ex-post. Over-investment can be reduced by simple restrictions on leverage. However, reducing leverage indirectly by tightening skin-in-the-game is never welfare improving.

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A Intermediary Problem

Rewrite the intermediaries' objective as follows:

$$\Pi_j = p[q_h R_1 I_{1,h,j} + y_{1,h,j}] + (1-p)[l(q_l R_1 I_{1,e,j} - c(I_{1,e,j}) + y_{1,e,j}) + (1-l)(q_l R_1 I_{1,ne,j} - c(I_{1,ne,j}) + y_{1,ne,j})] + q_l(\pi(b) - l)R_0((1-l)(I_{0,j} - S_{0,j} + T_{0,j}) + lT_{1,e,j} + (1-l)(T_{1,ne,j} - S_{1,ne,j})) - c(I_{0,j}) - \rho D_j.$$

Necessary conditions for an optimum are:

$$I_{0,j} : (1-p)q_l(\pi(b) - l)R_0(1-l) + (\lambda_{1,h,j} + \lambda_{1,e,j})R_0 + (\mu_{0,j} + \mu_{1,S,j})(1-a) - \lambda_{0,j} - c'(I_{0,j}) \leq 0 \quad (30)$$

$$T_{0,j} : (1-p)q_l(\pi(b) - l)R_0(1-l) + (\lambda_{1,h,j} + l(\lambda_{1,e,j} + \lambda_{1,ne,j}))R_0 + \mu_{1,T,j}(1-l) - \lambda_{0,j}p_0 \leq 0 \quad (31)$$

$$S_{0,j} : -(1-p)q_l(\pi(b) - l)R_0(1-l) - (\lambda_{1,h,j} + \lambda_{1,e,j})R_0 - (\mu_{0,j} + \mu_{1,S,j}) + \lambda_{0,j}p_0 \leq 0 \quad (32)$$

$$D_j : -\rho + \lambda_{0,j} - \rho\eta_{1,h,j} - \rho\eta_{1,e,j} - \rho\eta_{1,ne,j} \leq 0 \quad (33)$$

$$I_{1,h,j} : pq_h R_1 - \lambda_{1,h,j} \leq 0 \quad (34)$$

$$I_{1,e,j} : (1-p)lq_l R_1 - \lambda_{1,e,j} - (1-p)lc'(I_{1,e,j}) \leq 0 \quad (35)$$

$$T_{1,e,j} : (1-p)lq_l(\pi(b) - l)R_0 - \lambda_{1,e,j}p_1 \leq 0 \quad (36)$$

$$I_{1,ne,j} : (1-p)(1-l)q_l R_1 - \lambda_{1,ne,j} - (1-p)(1-l)c'(I_{1,ne,j}) \leq 0 \quad (37)$$

$$T_{1,ne,j} : (1-p)(1-l)q_l(\pi(b) - l)R_0 - \lambda_{1,ne,j}p_1 + \mu_{1,T,j} = 0 \quad (38)$$

$$S_{1,ne,j} : -(1-p)(1-l)q_l(\pi(b) - l)R_0 + \lambda_{1,ne,j}p_1 - \mu_{1,S,j} \leq 0 \quad (39)$$

$$y_{0,j} : -\lambda_{0,j} + \lambda_{1,h,j} + \lambda_{1,e,j} + \lambda_{1,ne,j} \leq 0 \quad (40)$$

$$y_{1,h,j} : p - \lambda_{1,h,j} + \eta_{1,h,j} = 0 \quad (41)$$

$$y_{1,e,j} : (1-p)l - \lambda_{1,e,j} + \eta_{1,e,j} = 0 \quad (42)$$

$$y_{1,ne,j} : (1-p)(1-l) - \lambda_{1,ne,j} + \eta_{1,ne,j} = 0 \quad (43)$$

It is straightforward to show that all budget and collateral constraints bind:

$$\lambda_{0,j}, \lambda_{1,h,j}, \lambda_{1,e,j}, \lambda_{1,ne,j}, \eta_{1,h,j}, \eta_{1,e,j}, \eta_{1,ne,j} > 0.$$

It is shown in Lemma 5 that when constrained at $t = 1$, $S_{0,j} = T_{0,j} = (1-a)I_{0,j}$. When unconstrained, intermediaries are indifferent over their choices of $T_{0,j}$ and $S_{0,j}$, so as discussed in the paper we simply assume they prefer to securitize all assets. To characterize the solution, we impose this and recast the intermediary problem accordingly. Focusing on the case in which $\mu_{1,T,j} > 0$, we use the constraints to rewrite the choice variables in terms of $I_{0,j}$, and $I_{1,e,j}$.

Choice variables given $I_{0,j}$, and $I_{1,e,j}$:

$$\begin{aligned}
y_{0,j} &= 0, \\
D_j &= I_{0,j} - w_{int}, \\
I_{1,h,j} &= R_0 I_{0,j} - \rho D_j, \\
&= (R_0 - \rho) I_{0,j} + \rho w_{int}, \\
y_{1,e,j} &= y_{1,ne,j} = y_{1,h,j} = \rho D_j, \\
T_{1,e,j} &= \frac{(aR_0 + (1-a)lR_0)I_{0,j} - \rho D_j - I_{1,e,j}}{p_1}, \\
&= \left(\frac{aR_0 + (1-a)lR_0 - \rho}{p_1} \right) I_{0,j} - \left(\frac{1}{p_1} \right) I_{1,e,j} + \left(\frac{1}{p_1} \right) \rho w_{int}, \\
T_{1,ne,j} &= -(1-l)(1-a)I_{0,j}, \\
I_{1,ne,j} &= ((1-a)(lR_0 + (1-l)p_1) - \rho)I_{0,j} + \rho w_{int},
\end{aligned}$$

The multipliers are determined by the first order conditions as follows:

$$\begin{aligned}
\lambda_{1,h,j} &= pq_h R_1, \\
\lambda_{1,e,j} &= (1-p)l(q_l R_1 - c'(I_{1,e,j})) = \frac{(1-p)lq_l(\pi(b) - l)R_0}{p_1}, \\
\lambda_{1,ne,j} &= (1-p)(1-l)(q_l R_1 - c'(I_{1,ne,j})), \\
\lambda_{0,j} &= \rho(\lambda_{1,h,j} + \lambda_{1,e,j} + \lambda_{1,ne,j}), \\
\eta_{1,h,j} &= \lambda_{1,h,j} - p, \\
\eta_{1,e,j} &= \lambda_{1,e,j} - (1-p)l, \\
\eta_{1,ne,j} &= \lambda_{1,ne,j} - (1-p)(1-l), \\
\mu_{0,j} &= \frac{lR_0 + (1-l)p_1}{p_1} \mu_{1,T,j}, \\
\mu_{1,T,j} &= \lambda_{1,ne,j} p_1 - (1-p)(1-l)q_l(\pi(b) - l)R_0, \\
&= p_1 \left[\lambda_{1,ne,j} - \frac{(1-l)}{l} \lambda_{1,e,j} \right].
\end{aligned}$$

Substituting the expressions above into the first order conditions on $I_{0,j}$ and $I_{1,e,j}$ and manipulating, we obtain the following three equations that characterize the optimal investment (we include $I_{1,ne,j}$ again for convenience).

$$(R_0 - \rho) \lambda_{1,h,j} + (aR_0 + (1-a)lR_0 - \rho) \lambda_{1,e,j} + (1-p)(1-l)(a + (1-a)l)q_l(\pi(b) - l)R_0 + (1-p)(1-l)(q_l R_1 - c'(I_{1,ne,j}))((1-a)(lR_0 + (1-l)p_1) - \rho) = c'(I_{0,j}), \quad (44)$$

$$q_l R_1 - \frac{q_l(\pi(b) - l)R_0}{p_1} = c'(I_{1,e,j}), \quad (45)$$

$$((1-a)(lR_0 + (1-l)p_1) - \rho)I_{0,j} + \rho w_{int} = I_{1,ne,j}. \quad (46)$$

The intermediaries' problem has a unique solution when the corresponding second-order conditions hold. As the objective is separable in $I_{0,j}$ and $I_{1,e,j}$, it suffices to show that $\frac{\partial^2 \pi_j}{\partial I_{0,j}^2} < 0$, $\frac{\partial^2 \pi_j}{\partial I_{1,e,j}^2} < 0$, which is true from the assumption that $c''(\cdot) > 0$.

The $t = 0, 1$ market-clearing conditions written in terms of (equilibrium) investments are:

$$I_{1,ne,j}^*(p_0, p_1) = ((lR_0 + (1-l)p_1)(1-a) - \rho) I_{0,j}^*(p_0, p_1) + \rho w_{int}, \quad (47)$$

$$I_{1,e,j}^*(p_0, p_1) = \left(R_0(l + a(1-l)) - \frac{(1-l)^2}{l}(1-a)p_1 - \rho \right) I_{0,j}^*(p_0, p_1) + \rho w_{int}. \quad (48)$$

These conditions have been simplified to show that prices are such that both late and early types invest available funds at time 1. Available funds for early (late) types is cash from time 0 investments, net of purchases (sales) and required collateral, $\rho(I_{0,j}^* - w_{int})$.

B Proofs

Proof of Lemma 1

Proof. Let $lR_0 \leq 1/\beta$. At $t = 0$, investors value securitized assets at $\beta\pi(r)R_0 = \beta lR_0 \leq 1 < p_0$. If $lR_0 > 1/\beta$, intermediaries can take on debt to purchase securitized assets and be guaranteed sufficient returns at $t = 1$ to cover the debt at $t = 2$. Thus, an equilibrium p_0 must be at least as large as gross returns on the time 0 asset to clear the market. Rewriting (1), gross returns are $lR_0 + (1-l)R_0[p + (1-p)q(\pi(b) - l)] > \beta lR_0$. Thus, investors do not purchase securitized assets in either case. \square

Proof of Lemma 2

Proof. Using $D_j = I_{0,j} - w_{int}$ from the solution to the intermediaries' problem we have

$$\frac{\partial D_j}{\partial \rho} = \frac{\partial I_{0,j}}{\partial \rho} = -\frac{\lambda_{1,h,j} + \lambda_{1,e,j} + \lambda_{1,ne,j}}{c''(I_{0,j})} < 0, \quad (49)$$

using (44) when intermediaries are unconstrained. In the constrained case, we have

$$\frac{\partial I_{0,j}}{\partial \rho} = -\frac{\lambda_{1,h,j} + \lambda_{1,e,j} + \lambda_{1,ne,j} - (1-p)(1-l)[c''(I_{1,ne,j})D_j((1-a)(lR_0 + (1-l)p_1) - \rho)]}{c''(I_{0,j}) + c''(I_{1,ne,j})(1-p)(1-l)((1-a)(lR_0 + (1-l)p_1) - \rho)^2},$$

where we have used (46) to obtain $I'_{1,ne,j} = ((1-a)(lR_0 + (1-l)p_1) - \rho)I'_{0,j} - (I_{0,j} - w_{int})$. Then, $\frac{\partial I_{0,j}}{\partial \rho} < 0$ when

$$(1-a)(lR_0 + (1-l)p_1) - \rho < \frac{\lambda_{1,h,j} + \lambda_{1,e,j} + \lambda_{1,ne,j}}{(1-p)(1-l)c''(I_{1,ne,j})(I_{0,j} - w_{int})}. \quad (50)$$

A sufficient condition for this is:

$$lR_0 - \rho < \frac{pq_h R_1 + (1-p)}{(1-p)(1-l)w_i c''((lR_0 - r)w_i + lR_0 w_{int})}. \quad (51)$$

In the constrained case, an increase in ρ not only reduces generally the available resources at $t = 1$, it also changes the value of resources for late types. When $(1-a)(l + (1-l)q_l(\pi(b) - l))R_0 - \rho > 0$, this value may in fact increase with ρ . When the above condition holds, it

ensures that this increase is dominated by the general decline resources. Hence, when this condition holds, demand for debt is always downward sloping.

To see that $D_j > 0$ when $\rho \leq E_\omega(\pi(\omega)) R_0 - c'(w_{int})$, first note that $I_{0,j} = D_j + w_{int}$. Then, using (30), and noting that $\lambda_{1,h,j} = pqR_1 > p$, $\lambda_{1,ne,j} \geq (1-p)(1-l)$, $\lambda_{1,e,j} \geq (1-p)l$, and $\mu_{1,T,j} \geq 0$, the marginal benefit of the first unit of debt is at least

$$\begin{aligned} & p(R_0 - \rho) + (1-p)l((a + (1-a)l)R_0 - \rho) + (1-p)(1-l)((1-a)lR_0 - \rho) \\ & + (1-p)(1-l)q_l(\pi(b) - l)R_0 - c'(w_{int}) \end{aligned} \quad (52)$$

$$= E_\omega(\pi(\omega)) R_0 - \rho - c'(w_{int}) \geq 0, \quad (53)$$

when $\rho \leq E_\omega(\pi(\omega)) R_0 - c'(w_{int})$. Thus, $D_j > 0$ and $I_{0,j} > w_{int}$.

To prove the second part of the Lemma, combine (33) with (41)-(43) to obtain $\lambda_{0,j} \leq \rho(\lambda_{1,h,j} + \lambda_{1,e,j} + \lambda_{1,ne,j})$. Then, using this in (40), the marginal benefit to holding reserves is $(-\rho + 1)(\lambda_{1,h,j} + \lambda_{1,e,j} + \lambda_{1,ne,j}) < 0$ as $\rho \geq \beta^{-1} > 1$.

To prove the next part of the Lemma, use (34) and (41) to obtain: $\eta_{1,h,j} = \lambda_{1,h,j} - p \geq pq_h R_h - p > 0$, (35) and (42) to obtain $\eta_{1,e,j} \geq (1-p)l(q_l R_1 - c'(I_{1,e,j})) > 0$, and (37) and (43) to get $\eta_{1,ne,j} \geq (1-p)(1-l)(q_l R_1 - c'(I_{1,ne,j})) > 0$, where all three strict inequalities follow from Assumption 2. \square

Proof of Lemma 3

Proof. When $p_1 < \underline{p}_1$, returns on purchasing assets strictly dominate investing, hence $I_{1,e,j} = 0$, and $T_{1,e,j} = \frac{aR_0 I_{0,j} + lR_0 T_{0,j} - \rho D_j}{p_1}$ via the budget constraint (3). When, $p_1 \geq \underline{p}_1$, combining (35) and (36), we obtain the desired characterization for $I_{1,e,j}$, and $T_{1,e,j}$ follows from (6). Furthermore,

$$I'_{1,e,j} = \frac{q(\pi(b) - l)R_0}{c''(I_{0,j})p_1^2} > 0. \quad (54)$$

Thus,

$$\frac{\partial T_{1,e,j}}{\partial p_1} = \frac{-I'_{1,e,j}p_1 - (aR_0 I_{0,j} + lR_0 T_{0,j} - rD_j - I_{1,e,j})}{p_1} < 0. \quad (55)$$

\square

Proof of Lemma 4

Proof. The proof is analogous to Lemma 3. \square

Proof of Lemma 5

Proof. Equations (35) and (36) yield

$$c'(I_{1,e}) = q_l \left(R_1 - \frac{(\pi(b) - l)R_0}{p_1} \right). \quad (56)$$

Similarly, using (37) and (38) we have

$$c'(I_{1,ne}) = q_l \left(R_1 - \frac{(\pi(b) - l)R_0 + \mu_{1,T,j}}{p_1} \right). \quad (57)$$

Hence, $\mu_{1,T,j} = 0 \Leftrightarrow I_{1,e,j} = I_{1,ne,j}$. Moreover, when $\mu_{1,T,j} > 0$, $c'(I_{1,ne,j}) < c'(I_{1,e,j}) \Rightarrow I_{1,e,j} > I_{1,ne,j}$ as $c(\cdot)$ is convex. Adding (31), and (32), we obtain $lR_0\lambda_{1,ne,j} - (1-l)R_0\lambda_{1,e,j} + (1-l)\mu_{1,T,j} \leq \mu_{0,j} + \mu_{1,S,j}$. Substituting expressions for $\lambda_{1,e,j}$, and $\lambda_{1,ne,j}$ from the first order conditions we can simplify as follows:

$$\mu_{1,T,j} \left(l \frac{R_0}{p_1} + (1-l) \right) = \mu_{0,j} + \mu_{1,S,j}. \quad (58)$$

Since the bracketed term in (58) is strictly positive, $\mu_{1,T,j} > 0 \Leftrightarrow \mu_{0,j} + \mu_{1,S,j} > 0$. Finally, to show that $\mu_{0,j} > 0$, and $\mu_{1,S,j} = 0$, we add (30), and (32), to obtain $[\lambda_{0,j}(p_0 - 1) - c'(I_{0,j})]/a = \mu_{0,j} + \mu_{1,S,j}$, when investment and sales are non-negative. This implies that $p_0 > 1 + c'(I_{0,j})/\lambda_{0,j} > 1$. Hence, it is profitable to invest and then sell assets at $t = 0$. As a result, $\mu_{0,j} > 0$, and therefore $\mu_{1,S,j} = 0$ since there are no assets to sell at period 1. \square

Proof of Lemma 6

Proof. The condition characterizing optimal $I_{0,j}$ is derived formally in Appendix A (see (44)). Noting that $S_{0,j} = T_{0,j}$, the expression for $D_{0,j}$ follows from the period 0 budget constraint. \square

Proof of Proposition 1

Proof. It suffices to show there is a unique p_1 that clears the asset market at $t = 1$. We focus on prices that satisfy $p_1 > \underline{p}_1 = q_l(\pi(b) - l)R_0/q_l R_1$, otherwise neither early or late types invest at $t = 1$. In an equilibrium with no investment at $t = 1$, excess demand is

$$\frac{1}{p_1}(lR_0I_{0,j} - \rho D_j), \quad (59)$$

where the bracketed term is aggregate resources net of debt obligations at $t = 1$. Thus, intermediaries invest exactly that amount which allows them to satisfy their debt requirements. This type of equilibrium will arise when investments at $t = 0$ are sufficiently more profitable than those at $t = 1$. With no investment at $t = 1$ there is no inefficiency in our model and thus we ignore these cases.

First, consider constrained equilibria at $t = 1$. As shown in Lemma 3, demand for assets at $t = 1$ is monotone decreasing. For a given $I_{0,j} > 0$, the quantity traded is $(1-a)(1-l)I_{0,j}$ and the unique price is simply determined by demand as characterized in Section A. We provide a condition on model primitives in Lemma 1.1 that is sufficient to ensure the existence of such a (unique) constrained equilibrium.

If the equilibrium is unconstrained, excess demand at $t = 1$ is given by

$$\frac{lR_0I_{0,j} - \rho D_j - I_1}{p_1}, \quad (60)$$

where $I_1 = I_{1,e,j} = I_{1,ne,j}$, since investments are equalized in the unconstrained case. Differentiating (60) with respect to p_1 gives

$$-\frac{I'_1}{p_1} - \left(\frac{lR_0I_{0,j} - \rho D_j - I_1}{p_1^2} \right) < 0, \quad (61)$$

as $I'_1 > 0$ and collateral constraints require $lR_0I_{0,j} - \rho D_j - I_1 \geq 0$. Thus excess demand is strictly decreasing in p_1 . Furthermore, demand exceeds supply at \underline{p}_1 , since $I_1 = 0$ at this price and resources must be positive for $t = 1$ investment to be non-zero. To show that excess demand is negative for some $p_1 < q_l(\pi(b) - l)R_0$, it suffices to show that demand is zero at this price. From the first order conditions on investment and asset purchases at $t = 1$, (35) and (36), we have

$$\frac{q_l(\pi(b) - l)R_0}{p_1} \leq q_l R_1 - c'(I_{1,e,j}). \quad (62)$$

At $p_1 = q_l(\pi(b) - l)R_0$, this collapses to $1 \leq q_l R_1 - c'(I_{1,e,j})$, which is strict by Assumption 2 and therefore demand for assets is zero. Given that excess demand is continuous, we conclude that an equilibrium in the asset market at $t = 1$ exists and is unique. \square

Proof of Corollary 1.1

Proof. When investment is positive, at an interior

$$S_1 = \frac{I_{1,ne,j} - (1 - a)lR_0I_{0,j} + \rho D_j}{p_1} \quad (63)$$

$$S'_1 = \underbrace{\frac{I'_{1,ne,j}}{p_1} - \frac{I_{1,ne,j}}{p_1^2}}_{\text{Investment motive}} + \underbrace{\frac{(1 - a)lR_0I_{0,j} - \rho D_j}{p_1^2}}_{\text{Collateral Motive}} = \frac{I'_{1,ne,j} - S_1}{p_1}. \quad (64)$$

The collateral motive is negative when $(1 - a)lR_0I_{0,j} - \rho D_j < 0$, so that late types must raise money to cover debt. If this effect is dominant, the supply curve is downward sloping.

We now show that supply can be upward sloping in the range where $I_{1,ne,j} > 0$. It is sufficient to show that $I'_{1,ne,j} - S_1 > 0$ can be positive at $S_1 = (1 - a)I_{0,j} - lT_{0,j} = (1 - a)(1 - l)I_{0,j}$, which is the maximum value of S_1 . Rewriting gives

$$\frac{q_l(\pi(b) - l)R_0}{c''(I_{1,ne,j})p_1^2} \geq (1 - a)(1 - l)I_{0,j}. \quad (65)$$

Using $p_1 < q_l(\pi(b) - l)R_0$ and $I_{0,j} < w_i + w_{int}$, we have the following sufficient condition

$$\frac{1}{c''(w_i(lR_0 - \rho) + w_{int}lR_0)} \geq q_l(\pi(b) - l)R_0(1 - a)(1 - l)(w_i + w_{int}), \quad (66)$$

if $c'''(\cdot) > 0$. The following condition is sufficient if $c'''(\cdot) < 0$

$$\frac{1}{c''(0)} \geq q_l(\pi(b) - l)R_0(1 - a)(1 - l)(w + w_{int}). \quad (67)$$

To complete the proof, we show that parameters exist such that the equilibrium may be constrained. First, posit an unconstrained equilibrium in which excess demand is zero. From the first order conditions we have

$$p_1 = \frac{q_l(\pi(b) - l)R_0}{q_l R_1 - c'(I_{1,j})}, \quad (68)$$

where $I_{1,j} = I_{1,e,j} = I_{1,ne,j} = lR_0I_{0,j} - \rho D_j$, since the sales constraint is slack by assumption. Substituting this into the individuals' supply function gives the equilibrium quantity

$$S_{1,j} = \frac{alR_0I_{0,j}(q_lR_1 - c'(I_{1,j}))}{q_l(\pi(b) - l)R_0}. \quad (69)$$

If $S_{1,j} > (1-l)(1-a)I_{0,j}$, then the equilibrium must be constrained. The time 1 asset is assumed to be positive NPV, so that $q_lR_1 - c'(I_{1,j}) > 1$. Using this and simplifying gives

$$(\pi(b) - l) \frac{(1-l)}{l} \frac{(1-a)}{a} \leq \frac{1}{q_l}. \quad (70)$$

□

Proof of Proposition 2

Proof. We first outline the salient features of solution to the planning problem described in (22)-(27). It is straightforward to show that all budget and collateral constraints bind, that no reserves are held at $t = 0$ and the interior solution is unique since $-c''(I_0) < 0$. Combining the first order conditions on $t = 1$ investment gives

$$(1-p)(q_lR_1 - c'(I_{1,e})) = (1-p)(q_lR_1 - c'(I_{1,e})) \implies I_{1,e} = I_{1,ne} = I_1. \quad (71)$$

Furthermore,

$$I_{1,h} = (R_0 - \rho)I_0 + \rho w_{int}, \quad (72)$$

$$I_{1,e} = I_{1,ne} = (lR_0 - \rho)I_0 + \rho w_{int}, \quad (73)$$

where we have used $D = I_0 - w_{int}$. Investments at $t = 0$ and $t = 1$ are thus related as follows:

$$(1-p)(lR_0 - \rho)(q_lR_1 - c'(I_1)) + p(R_0 - \rho)(q_hR_1) = c'(I_0). \quad (74)$$

The planner equates the marginal cost of investment at $t = 0$ with the marginal benefit of investment across the states at $t = 1$, where investments are equalized across intermediary types in the low state.

Contingent Securities

We now introduce contingent securities traded at $t = 0$, conditional on individual type at $t = 1$, when $\sigma = l$ (there is no gains from trading securities that pay off when $\sigma = h$ since all intermediaries are identical in this state). Further, there is no motive for trade at $t = 1$ and we ignore this possibility. The security pays the owner one unit, conditional on $\sigma = l$ and the realization of late returns. Denote by ζ_{ne} and ζ_e the quantities of this security purchased or sold, and ρ_0 the corresponding price. The intermediaries' problem is:

$$\begin{aligned} \Pi_j = & p[q_hR_1I_{1,h,j} + y_{1,h,j}] + (1-p)l[q_lR_1I_{1,e,j} - c(I_{1,e,j}) + y_{1,e,j}] \\ & + (1-p)(1-l)[q_lR_1I_{1,ne,j} - c(I_{1,ne,j}) + y_{1,ne,j}] - c(I_{0,j}) - \rho D_j, \end{aligned} \quad (75)$$

subject to:

$$(\lambda_{0,j}) \quad I_{0,j} + \rho_0(\zeta_{ne,j} - \zeta_{e,j}) + y_{0,j} \leq w_{int} + D_j, \quad (76)$$

$$(\lambda_{1,h,j}) \quad I_{1,h,j} + y_{1,h,j} \leq R_0 I_{0,j} + y_{0,j}, \quad (77)$$

$$(\lambda_{1,e,j}) \quad I_{1,e,j} + y_{1,e,j} \leq R_0 I_{0,j} - (1-l)\zeta_{e,j} + y_{0,j}, \quad (78)$$

$$(\lambda_{1,ne,j}) \quad I_{1,ne,j} + y_{1,ne,j} \leq -l\zeta_{ne,j} + y_{0,j}, \quad (79)$$

$$(\eta_{1,h,j}) \quad \rho D_j \leq y_{1,h,j}, \quad (80)$$

$$(\eta_{1,e,j}) \quad \rho D_j \leq y_{1,e,j}, \quad (81)$$

$$(\eta_{1,ne,j}) \quad \rho D_j \leq y_{1,ne,j}. \quad (82)$$

All budget constraints bind, $y_{0,j} = 0$, and all collateral constraints will bind, so that $y_{1,h,j} = y_{1,e,j} = y_{1,ne,j} = \rho D_j$. We can then rewrite the objective and obtain the following necessary optimality conditions:

$$I_{0,j} : pq_h R_1(R_0 - \rho) + \lambda_{1,e,j}(R_0 - \rho) + \lambda_{1,ne,j}(-\rho) = 0, \quad (83)$$

$$I_{1,e,j} : (1-p)l[q_l R_1 - c'(I_{1,e,j})] = \lambda_{1,e,j}, \quad (84)$$

$$I_{1,ne,j} : (1-p)(1-l)[q_l R_1 - c'(I_{1,ne,j})] = \lambda_{1,ne,j}, \quad (85)$$

$$\zeta_{ne,j} : pq_h R_1(-\rho p_0) + \lambda_{1,e,j}(-\rho p_0) + \lambda_{1,ne,j}(l - \rho p_0) = 0, \quad (86)$$

$$\zeta_{e,j} : pq_h R_1(\rho p_0) + \lambda_{1,e,j}(\rho p_0 - (1-l)) + \lambda_{1,ne,j}(\rho p_0) = 0. \quad (87)$$

Using (86) and (87), we obtain $\lambda_{1,e,j}/l = \lambda_{1,ne,j}/(1-l)$, which substituted into the first order conditions for $t = 1$ investment yield $I_{1,e,j} = I_{1,ne,j} = I_{1,j}$. Thus, trading securities at $t = 0$ permits agents to equate the marginal returns to investment at $t = 1$. Write the foc for $I_{0,j}$ as follows:

$$pq_h R_1(R_0 - \rho) + (1-p)[q_l R_1 - c'(I_{1,j})](lR_0 - \rho) = c'(I_{0,j}). \quad (88)$$

This is identical to (74), hence the market allocation when Arrow securities are traded is Pareto efficient. \square

Proof of Proposition 3

Proof. When $a = 0$ intermediaries are never constrained at $t = 1$. Suppose they are constrained, then by Lemma 5, they are constrained at $t = 0$. As $a = 0$, this implies that $S_{0,j} = I_{0,j}$, but then at $t = 1$ early and late intermediaries have the same resources. Thus, there is no motive for trade at $t = 1$, and therefore no asset sales and hence intermediaries are never constrained. Clearly, this implies that $I_{1,e,j} = I_{1,ne,j} = I_1$.

The necessary conditions for an optimum to the intermediaries' problem can be reduced to the following:

$$I_{0,j} : (1-p)q_l(\pi(b) - l)R_0(1-l) - c'(I_{0,j}) + [\lambda_{1,h,j} + \lambda_{1,e,j} + \lambda_{1,ne,j}](R_0 - \rho) = 0, \quad (89)$$

$$I_{1,e,j} : (1-p)lq_l R_1 - \lambda_{1,e,j} - (1-p)lc'(I_1) = 0, \quad (90)$$

$$I_{1,ne,j} : (1-p)(1-l)q_l R_1 - \lambda_{1,ne,j} - (1-p)(1-l)c'(I_1) = 0. \quad (91)$$

Substituting (90) and (91) into (89), we obtain (74) from the solution to the Planner's problem. \square

Proof of Proposition 4

Proof. Differentiating the Lagrangian for the intermediaries' problem we have:

$$\frac{d\Pi_j^*}{dI_0} = \frac{dp_1^*}{dI_0} \left[\frac{\mu_{1,T,j}^*}{p_1^*} (1-l)(1-a)I_0^* \right], \quad (92)$$

where we use the market clearing condition $lT_{1,e,j}^* = -(1-l)T_{1,ne,j}^*$. Since the price effect in (92) is non-zero, the equilibrium is inefficient whenever $\mu_{1,T,j}^* > 0$.

To characterize the nature of the inefficiency, note that from (92), $\text{sign}(d\Pi_j^*/dI_0) = \text{sign}(dp_1^*/dI_0)$. Multiplying the market clearing condition at $t = 1$ by p_1^* and noting that $T_{1,ne,j}^* = -(1-l)T_{0,j}^* = -(1-l)(1-a)I_{0,j}^*$ due to the binding sales constraint implies that $p_1 l T_{1,e}^* = p_1 (1-l)^2 (1-a) I_{0,j}^*$. Differentiating with respect to I_0 , and noting that $S_{0,j}^* = T_{0,j}^*$ gives:

$$\frac{dp_1^*}{dI_0} = \frac{l((l+a(1-l))R_0 - 1/\beta) - (1-l)^2(1-a)p_1^*}{l \frac{dI_{1,e}^*}{dp_1} + (1-l)^2(1-a)I_0^*}. \quad (93)$$

The denominator is strictly positive since $\frac{dI_{1,e}^*}{dp_1} = \frac{q_l(\pi(b)-l)R_0}{c''(I_{1,e,j}^*)p_1^{*2}} > 0$ as $c'' > 0$. Thus, the sign of dp_1^*/dI_0 is determined solely by the numerator. First, we characterize parameters to support the under-investment case. Under-investment occurs when

$$p_1^* < \frac{l((l+a(1-l))R_0 - 1/\beta)}{(1-l)^2(1-a)}. \quad (94)$$

From Lemma 5, we know that $I_{1,ne,j}^* < I_{1,e,j}^*$ when late sellers are constrained. Using (47) and (48), we have the following upper bound on the price in a constrained equilibrium:

$$p_1^* < \bar{p}_c = \frac{alR_0}{(1-a)(1-l)}. \quad (95)$$

Therefore, a sufficient condition for (94) is

$$\frac{alR_0}{(1-l)(1-a)} < \frac{l((l+a(1-l))R_0 - 1/\beta)}{(1-l)^2(1-a)} \iff lR_0 > 1/\beta, \quad (96)$$

which is the expression in Proposition 4.

Over-investment occurs when the inequality in (94) is reversed. Positive investment at $t = 1$ requires $p_1^* \geq \underline{p}_1 = (\pi(b)-l)R_0/R_1$, and thus a sufficient condition for over-investment is

$$\frac{(\pi(b)-l)R_0}{R_1} > \frac{l((l+a(1-l))R_0 - 1/\beta)}{(1-l)^2(1-a)}. \quad (97)$$

Manipulating gives

$$\frac{R_0}{R_1}(\pi(b)-l) \frac{(1-l)}{l} \frac{(1-a)}{a} > \frac{lR_0 - 1/\beta + a(1-l)R_0}{a(1-l)}. \quad (98)$$

Assume that

$$(\pi(b) - l) \frac{(1-l)}{l} \frac{(1-a)}{a} = \frac{1}{q_l}, \quad (99)$$

which from (70), ensures a constrained equilibrium. Substituting (99) into (98) we have

$$1/\beta - lR_0 > a(1-l)R_0 \left(1 - \frac{1}{q_l R_1}\right). \quad (100)$$

Since $1 - 1/q_l R_1 < 1$, the condition in Proposition 4 follows. \square

Proof of Proposition 5

Proof.

$$\frac{d\Pi_j^*}{da} = \frac{\mu_{1,T,j}^* I_{0,j}^*}{p_1^*} \left[(1-a)(1-l) \frac{dp_1^*}{da} - lR_0 - (1-l)p_1^* \right], \quad (101)$$

where we have used (58), and $T_{0,j}^* = S_{0,j}^* = (1-a)I_{0,j}^*$. We ignore the price effects arising from changes in p_0^* , since changes to p_0^* have no welfare implications at the equilibrium. This is because all intermediaries have net zero sales at $t = 0$, thus changes in p_0^* do not alter the budget and thus do not affect investment.

The $t = 1$ market-clearing condition, $lT_{1,e}^* = (1-l)T_{1,ne}^*$ can be expressed as:

$$l \frac{((a + (1-a)l)R_0 - 1/\beta) I_{0,j}^* + w_{int}/\beta - I_{1,e,j}^*}{p_1^*} = (1-l)^2(1-a)I_{0,j}^*. \quad (102)$$

Differentiating both sides with respect to a and solving for $\frac{dp_1^*}{da}$ we have:

$$\frac{dp_1^*}{da} = \frac{lR_0 + (1-l)p_1^*}{(1-l)(1-a)} \cdot \left[\frac{(1-l)(1-a) \left((1-l)I_{0,j}^* - \frac{M}{lR_0 + (1-l)p_1^*} \frac{\partial I_{0,j}^*}{\partial a} \right)}{M \frac{\partial I_{0,j}^*}{\partial p_1} + \frac{lq_l(\pi(b)-l)R_0}{p_1^{*2} c''(I_{1,e,j}^*)} + (1-l)^2(1-a)I_{0,j}^*} \right], \quad (103)$$

where we have used $\frac{dI_{1,e,j}^*}{da} = \frac{q_l(\pi(b)-l)R_0}{p_1^{*2} c''(I_{1,e,j}^*)} \frac{dp_1^*}{da}$, and

$$\frac{\partial I_{0,j}^*}{\partial a} = \frac{(lR_0 + (1-l)p_1)(1-p)(1-l) (c'_{ne} - c'_e + c''_{ne} M_{ne} I_{0,j}^*)}{(1-p)(1-l)c''_{ne} M_{ne}^2 + c''_0}, \quad (104)$$

$$\begin{aligned} \frac{\partial I_{0,j}^*}{\partial p_1} &= \frac{-\frac{M_e(1-p)lq_l(\pi(b)-l)R_0}{p_1^2} + (1-p)(1-l)^2(1-a)(q_l R_1 - c'_{ne} - c''_{ne} M_{ne} I_{0,j}^*)}{(1-p)(1-l)c''_{ne} M_{ne}^2 + c''_0}, \\ &= \frac{\frac{\mu_{1,T,j}(1-a)(1-l)}{p_1} + \frac{M\lambda_{1,ne,j}}{lp_1} - c''_{ne} M_{ne} I_{0,j}^*}{(1-p)(1-l)c''_{ne} M_{ne}^2 + c''_0}, \end{aligned} \quad (105)$$

$$M = (1-l)^2(1-a)p_1 - l((a + (1-a)l)R_0 - \beta^{-1}), \quad (106)$$

$$M_e = aR_0 + (1-a)lR_0 - \beta^{-1}, \quad (107)$$

$$M_{ne} = (1-a)(lR_0 + (1-l)p_1) - \beta^{-1}. \quad (108)$$

Then, $\frac{d\Pi_j^*}{da} > 0$ when the following is true

$$\frac{dp_1^*}{da} > \frac{lR_0 + (1-l)p_1^*}{(1-a)(1-l)} \iff \Omega \equiv \left[\frac{(1-l)(1-a) \left((1-l)I_{0,j}^* - \frac{M}{lR_0 + (1-l)p_1^*} \frac{\partial I_{0,j}^*}{\partial a} \right)}{M \frac{\partial I_{0,j}^*}{\partial p_1} + \frac{lq_l(\pi(b)-l)R_0}{p_1^{*2}c''(I_{1,e,j}^*)} + (1-l)^2(1-a)I_{0,j}^*} \right] > 1. \quad (109)$$

Using $q_l R_1 - \frac{q_l(\pi(b)-l)R_0}{p_1^*} = c'(I_{1,e,j}^*)$ from the first order conditions, rewrite Ω as

$$\Omega = \frac{\epsilon}{\epsilon + \phi}, \quad (110)$$

where

$$\epsilon = (1-l)^2(1-a)I_{0,j}^* + \frac{(1-p)(1-l)^2(1-a)M}{\delta} \left(c'_e - c'_{ne} - c''_{ne} M_{ne} I_{0,j}^* \right), \quad (111)$$

$$\phi = \frac{q_l(\pi(b)-l)R_0}{p_1^*} \left(\frac{(1-p)(1-l)^2(1-a)M}{\delta} + \frac{l}{p_1^* c''(I_{1,e,j}^*)} - \frac{M M_e (1-p)l}{\delta p_1^*} \right), \quad (112)$$

$$\delta = (1-p)(1-l)c''_{ne} M_{ne}^2 + c''_0 > 0. \quad (113)$$

Manipulation gives

$$\phi = \frac{(1-p)q_l(\pi(b)-l)R_0}{p_1 \delta} \left(\frac{M^2}{p_1} + \frac{l\delta}{p_1 c''(I_{1,e,j}^*)} \right) > 0. \quad (114)$$

$M > 0$ is simply the condition for over-investment, which is derived in the proof of Proposition 4. Further, $M_{ne} < 0$ in the over-investment case. To see this, note that M_{ne} is strictly increasing in p_1 . Evaluate M_{ne} at the maximum constrained price $\bar{p}_c = alR_0/(1-l)(1-a)$ described in (95). This gives $M_{ne}(\bar{p}_c) = lR_0 - \beta^{-1}$, which must be negative in the over-investment case as shown in the proof of Proposition 4. Thus, both ϵ and ϕ are strictly positive and $\Omega < 1$, implying the effect of the change is negative. \square