Four colours and the visual separation of adjacent areas: lessons from mapping and ancient paintings

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> The *four-color problem* is very simply stated and has to do with the coloring of maps. Naturally, when coloring a map we wish to color neighboring countries differently so that we can tell them apart. So how many colors do we need to color the entire map?[1]

This question was first posed about 150 years ago by the mathematician Francis Guthrie, and the history of providing the answer is an intriguing one. Writing over half a century ago, the mathematician and historian of mathematics, Kenneth May, provided one of the earliest of several historical accounts in 'The Origin of the Four-color Conjecture', concluding 'Rarely is a mathematical invention the work of a single individual, and assigning names to results is generally unjust. In this case, however, it would seem that the four-color conjecture belongs uniquely to Francis Guthrie and could fairly be called Guthrie's problem.'[2] That the answer to the problem is four colours was initially reportedly 'found by experience',[3] probably just two years after the question was first raised in 1854. But it took over a century for the work of many highly talented mathematicians and many hours of computer time to prove indeed that *Four Colors Suffice*.[1] The proof was published in 1996, although even then with a mild disclaimer.[4]

But the four-colour problem represents more than solely a mathematical conundrum dealing with the delineation of boundaries and separateness and involving a limited palette, for the use of such a limited palette has featured in an entirely different context in the visual arts dating back many centuries. Yet

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[1] This statement of the variously termed four-colour problem, conjecture, or theory opens the comprehensive and entertaining history and discussion of the subject by Robin Wilson in *Four Colors Suffice: How the Map Problem was Solved*, Princeton, NJ: Princeton University Press, 2014, 1. This monograph also includes an extensive bibliography. I am indebted to the late Dr Robin Willison for drawing my attention to the four-colour problem.

[2] Kenneth May, 'The Origin of the Four-color Conjecture', *Isis*, vol. 56, 1965, 346-348.

[3] Brendan McKay, 'A Note on the History of the Four-colour Conjecture', *Journal of Graph Theory*, 2013, vol. 72, 361-363.

[4] The paper by Neil Robertson, Daniel Sanders, Paul Seymour, and Robin Thomas, 'A New Proof of the Four-colour Theorem', appeared in *Electronic Research Announcements of the American Mathematical Society*, 1996, vol. 2, 17-25. It had appeared earlier online as a '…a brief summary of a new proof', and 'Updated 13 November 1995' by some of the (unidentified) authors, at http://www.math.gatech.edu/~thomas/FC/fourcolor.html. The two versions are identical. The proof may still not be definitive; the authors acknowledge that 'our 'proof' is not a proof in the traditional sense, because it contains steps that can never be verified by humans…We concede, however, that verifying a computer program is much more difficult than checking a mathematical proof of the same length.'

although the mathematical and artistic spheres appear seemingly unrelated, a story recounted by Pliny and cited below similarly features just three or four colours – but here employed for the depiction and separation of individual lines rather than different areas which relate to the four-colour mapping conjecture of today.

Comparatively little is known about the prehistorical artistry of the Upper Palaeolithic period, but just four colours – yellow, red, black and brown – are seen in the cave paintings at Lascaux created around 16,000 BC.[5] In comparison, far more is known about the use of colour and its importance to artists many millennia later in the classical world of ancient Greece. During that period many artists indeed used very few colours, these providing all that was needed as a method of distinguishing different areas in a painting.

In those classical times, absolute simplicity was favoured by artists, and four colours were thought to echo philosophical and metaphysical entities which could variously comprise four geometrical solids, the four elements, the four seasons, and the four directions in the heavens.[6] Whatever background explanations there may have been, this use of a restricted colour palette in ancient Greece was a subject discussed centuries later – in somewhat conflicting accounts – by the Roman writers Cicero and Pliny the Elder. Thus Cicero, writing about beauty in painting, notes that '…in the works of Zeuxis, Polygnotus, Timanthes, and several others masters who confined themselves to the use of four colours, we commend the air and the symmetry of their figures; but in Aetion, Nicomachus, Protogenes, and Apelles, every thing is finished to perfection.'[7] And Pliny, having described that the Greek painter Apelles had painted pictures including *Alexander the Great Holding a Thunderbolt*, continues: 'readers must remember that all these effects were produced

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[5] See http://archeologie.culture.fr/lascaux/en.

[6] The subject is discussed in the chapter 'The Four-Color Palette of the Greeks' in Vincent Bruno, *Form and Color in Greek Painting*, New York: W. W. Norton & Co., 1977, 53-59. Another authority, John Gage, in 'A Locus Classicus of Colour Theory: The Fortunes of Apelles', *Journal of the Warburg and Courtauld Institutes*, 1981, vol. 44, 1-26, notes that 'Pliny's circumstantial account, although it is not the only one, has provided the basis of all subsequent discussions of the four-colour palette' (p. 2). Gage then considers the subject at length, including the types of pigments and their combinations, the idea of the primaries [i.e. primary colours], and the historical developments of these subjects. He also acknowledges the important contribution that Bruno's book published just four years earlier made to the historical discussion. I am grateful to A. A. Donohue for drawing my attention to the pertinent study of ancient Greek 'Four Elements/Four Colors philosophy' in J. L. Benson's monograph *Greek Color Theory and the Four Elements*: *A Cosmological Interpretation*, Amherst, Mass.: University of Massachusetts Amherst Libraries, 2000. Benson provides an exhaustive and provocative overview, including much more recent aspects of colour theory.

[7] Cicero, *Brutus,* transl. G L Hendrickson, xviii, 69-71, Loeb Classical Library 342, Cambridge, MA: Harvard University Press & London: William Heinemann, 1952. For the definitive reference work discussing this text, see Harald Mielsch and Lauri Lehmann in: *Der neue Overbeck. Die antiken Schriftquellen zu den bildenden Künsten der Griechen* [DNO], Sascha Kansteiner, Klaus Hallof, Lauri Lehmann, Bernd Seidensticker, and Klaus Stemmer, eds., volume IV, entry no. 2944, Berlin and Boston: De Gruyter, 2015.

by four colours.'[8] Four colours thus appear fundamental to the method of painting of the ancients, and both Cicero and Pliny make particular mention of Apelles, perhaps the greatest of the classical Greek painters, who flourished in the 4th century BC.

Which four colours these artists, and notably Apelles, used remains a very confused issue. It is likely that they employed what would now be called primary colours 'by means of which all the innumerable variations of color and tone in nature can be obtained by a process of mixing and blending…[and which] cannot be obtained by mixing other colors'[9]; these would typically comprise red, yellow and blue. But a problem arises: Pliny reports 'Four colours only were used by the illustrious painters' who included Apelles amongst others, these colours being white, yellow, red and black.[10] While white and black would then have been included as colours with their equivalence of light and dark, a conclusion which would not find favour today, the omission of blue is both striking and puzzling, although it had been suggested that 'a blue-black pigment' could rather have been used as a darkening agent 'capable of producing a high degree of nuance in chiaroscuro and in the variety of reds and yellows, browns and ochres obtainable.'[11] Nevertheless, is that the explanation for the omission of blue?

The problem was resolved by Sir Humphry Davy: '…it is not unlikely that Pliny was misled by an imperfect recollection of a passage in Cicero… [and] Pliny himself describes with enthusiasm the *Venus*…of Apelles: and in this picture the sea was represented, which required azure.'[12] Thus blue may indeed have been the fourth colour, in addition to yellow, red and white – black being reserved for outlines, for instance on Greek pottery.

Returning to mapping, the role of four colours lies in their most economical ability to depict and separate multiple adjacent areas. While this role might seem far removed from the classical painting techniques described above, an enchanting story establishes that the identical function of depiction and separation by means of colours occurred two and a half millennia ago in ancient Greece.

This well-known story told by Pliny recounts that Apelles wished to see the works of Protogenes, another famous artist. On visiting him, however, it is related that

> The artist was not there but there was a panel of considerable size on the
> easel prepared for painting, which was in the charge of a single old woman.
> In answer to his enquiry, she told him that Protogenes was not at home, and

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[8] Pliny the Elder, *Natural History*, transl. Harris Rackham, volume IX, book xxxv, 92, Loeb Classical Library 394, Cambridge, MA & London: Harvard University Press, 1952. For the definitive reference work discussing this text, see Mielsch and Lehmann, DNO (op. cit), entry no. 2942.

[9] Bruno, *Form and Color*, 54.

[10] Pliny, *Natural History*, 50.

[11] Bruno, *Form and Color*, 96.

[12] Sir Humphry Davy, 'Some Experiments and Observations on the Colours used in Painting by the Ancients', *Philosophical Transactions of the Royal Society*, 1815, vol. 105, 97-124, especially 123.

asked who it was she should report as having wished to see him. 'Say it was this person,' said Apelles, and taking up a brush he painted in colour across the panel an extremely fine line; and when Protogenes returned the old woman showed him what had taken place. The story goes that the artist, after looking closely at the finish of this, said that the new arrival was Apelles, as so perfect a piece of work tallied with nobody else; and he himself, using another colour, drew a still finer line exactly on the top of the first one, and leaving the room told the attendant to show it to the visitor if he returned and add that this was the person he was in search of; and so it happened; for Apelles came back, and, ashamed to be beaten, cut the lines with another in a third colour, leaving no room for any further display of minute work.[13]

The colours described in this story must therefore have numbered three, or possibly four if one includes the colour of the background panel.

The relevance of the story lies here in the conclusion that, despite colours being universal and all pervasive, surprisingly only four are necessary to define and separate different pictorial elements and their boundaries − whether those elements comprise areas on a map as was realised 150 years ago, or components of a picture as was realised at least two and half millennia ago. Furthermore, it is evident that defining and separating by means of four colours can be achieved whether those colours appear as the finest of lines or in two-dimensional areas. Proving that four colours suffice, however, remains a pursuit of mathematicians rather than artists or historians.

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[13] Pliny, *Natural History*, 81-83. Leon Battista Alberti, in *On Painting* transl. Cecil Grayson, London: Penguin Books, 1972, confirms that 'Some people express astonishment that the ancient painters Polygnotus and Timanthes used only four colours' (p. 82). Alberti comments too on the famous competition as to who could draw the 'finest possible, almost invisible lines, like those they say the painter Apelles used to practise and vie with Protogenes at drawing" (p. 65).